Strain-Rate Dependant Damage Material Model of Layered Fabric Composites with Delamination Prediction for Impact Simulations

Sylvain Treutenaere[1], Franck Lauro[1], Bruno Bennani[1], Tsukatada Matsumoto[2], Ernesto Mottola[2]

[1]University of Valenciennes and Hainaut Cambrésis, LAMIH, UMR CNRS 8201, 59313 Valenciennes, France
[2]TOYOTA MOTOR EUROPE, 1140 Brussels, Belgium

1 Introduction

The use of carbon fabric reinforced polymers (CFRP) in the automotive industry increased very significantly due to their high specific stiffness and strength, their great energy absorption as well as the reduced manufacturing cost.
The behaviour understanding and modelling of these materials become essential for their implementation into the design loop, needed for the deployment on mass-produced vehicles. In order to ensure the protection of pedestrians and drivers/passengers in case of collision with a CFRP panel, a model dedicated to the finite element analysis (FEA) of impacts is needed.
The nonlinear material behaviour which leads to differences in the impact response of composites is attributed to fibre failure, intra- and interlaminar matrix cracking, fibre-matrix debonding and strain rate sensitivity of the matrix. Additionally, the textile composite materials are capable of large shearing prior to the ultimate failure due to the sliding and reorientation of yarns. The modelling of all these phenomena is essential to describe the impact behaviour of layered fabric composites.
This study is focused on the matrix damage (intralaminar and interlaminar) modelling in a finite strain framework.
The intralaminar matrix damage model relies on a close version of the Onera Damage Model proposed by Marcin [1]. Marcin adapted and extended the micromechanics based Continuum Damage Mechanic model proposed by Chaboche [2] to the fabric reinforced materials for implicit simulations.
The spectral viscoelastic model used in the Onera Damage Model is replaced by a generalized Maxwell model. It has been successfully extended and used in finite strain for anisotropic materials [3]. This present damageable viscoelastic model is then expressed in the finite strain framework.
The consideration of the interlaminar damage in finite element analysis is nowadays a challenge. The classical framework is to use cohesive elements associated to specific cohesive laws in the ply interface layers. However this method is time-consuming and does not allow the formulation of the dependence between the intralaminar and the interlaminar damages. Moreover, the number of elements through the thickness of the layered material has to be expanded to approach the real transverse strain distribution. Another solution is to recompute the strain distribution by using a higher-order zigzag formulation. Kim and Cho [4] have proposed a formulation with five degrees of freedom and which takes into account interfacial imperfections. By ensuring an energy balancing between this formulation and the formulation used by the element, it is possible to recover a realistic displacement field through the thickness of the layered materials [5]. This theory has been implemented in a user material subroutine in LS-Dyna.

2 Continuum matrix damage model

Experimentally the matrix cracks are observed to be oriented in the directions of reinforcement or transverse to them. As the *Onera Damage Model MicroStructure* (ODM_MS) [1] is based on this assumption, the present model relies on a close version of the ODM_MS.

2.1 Constitutive relation

The model is formulated in strain-space to ensure a good efficiency for FEA. The constitutive relation, by using the Voigt notation, is defined as follows

$$\sigma = \tilde{\boldsymbol{C}}^{\boldsymbol{m}} : \varepsilon - \boldsymbol{C}^{\boldsymbol{0}} : (\varepsilon^r + \varepsilon^s) \qquad (1)$$

where σ and ε are respectively the infinitesimal stress and strain tensor. $\boldsymbol{C^0}$ is the initial stiffness tensor and $\widetilde{\boldsymbol{C}}^{\boldsymbol{m}}$ is the effective stiffness tensor.

The stored strains ε^s are needed in order to avoid the discontinuity of the (σ, ε) response for bi-axial loading, and to ensure the recovery of the initial stiffness after the damage deactivation.

The evolution of the residual strains ε^r is dependent on the damage evolution.

2.2 Effective stiffness tensor

The damage is introduced by adding additional compliance to the initial compliance tensor $\boldsymbol{S}^0$. The effective stiffness tensor is thus defined with

$$(\widetilde{\boldsymbol{C}}^{\boldsymbol{m}})^{-1} = \boldsymbol{S}^0 + \Delta \boldsymbol{S}^{\boldsymbol{m}}. \tag{2}$$

The additional compliance tensor due to the matrix damage is given by

$$\Delta \boldsymbol{S}^{\boldsymbol{m}} = \sum_i \eta_i^m d_i^m \boldsymbol{H}_{\boldsymbol{i}}^{\boldsymbol{m}} \tag{3}$$

where $\boldsymbol{H}_{\boldsymbol{i}}^{\boldsymbol{m}}$ is the compliance tensor associated with the damage variable d_i^m. η_i^m represents the crack closure index which varies from 0 (closed crack) to 1 (opened crack).

2.3 Damage evolution

In order to describe separately the different fracture modes, two driving forces affect the evolution of each damage variable: the normal y_i^{nm} and the tangential y_i^{tm} to the damage directions. The thermodynamic forces so defined are dependent on the effective strain tensor, which corresponds to the positive part of the spectral decomposition of the strain tensor.

Therefore, the damage criterion is defined as follows

$$F_i^m = f_i^{nm}(y_i^{nm}) + f_i^{tm}(y_i^{tm}) - d_i^m \leq 0 \tag{4}$$

with f the cumulative distribution function of Weibull.

This formulation — compared to a standard frame where the thermodynamic forces are based on the derivative of the Helmholtz free energy — makes easier the parameter identification, retains the explicitness of the formulation and ensures the thermodynamic coherence.

3 Viscoelastic damageable continuum model

The generalised Maxwell model is well adapted to the explicit finite element method because of the direct dependence of the total stress with the strain. It is symbolically represented by a spring of stiffness $\boldsymbol{C}_\infty$ paralleled with N Maxwell elements (Fig.1).

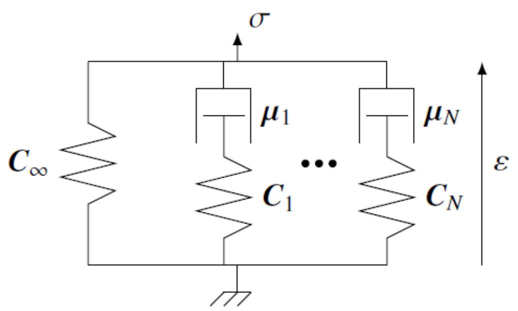

Fig.1: Generalised Maxwell Model

Hence, the total stress, provided by the coupling between the above mentioned matrix damage and the generalised Maxwell viscoelastic models, is given by

$$\sigma = \sigma_\infty + \sum_i \widetilde{h}_j \tag{5}$$

where σ_∞ is the long-time stress, defined by means of the ODM_MS with the equation (3), and $\tilde{h}_j$ is the effective viscoelastic stress of the j-th branch. This tensor is determined thanks to the Boltzman superposition principle and is expressed in an iterative form [3] by

$$\tilde{h}_j(t^{n+1}) = \exp(-\frac{\Delta t}{\tau_j}) \cdot \tilde{h}_j(t^n) + \tilde{\boldsymbol{C}}_j^{\boldsymbol{m}} \frac{1-\exp(-\Delta t/\tau_j)}{\Delta t/\tau_j} \Delta\varepsilon \tag{6}$$

with $\Delta t = t^{n+1} - t^n$, $\Delta\varepsilon = \varepsilon(t^{n+1}) - \varepsilon(t^n)$ and $\tilde{\boldsymbol{C}}_j^{\boldsymbol{m}}$ the effective viscoelastic stiffness tensor. This tensor is defined in a same manner as the effective stiffness tensor presented in the subsection 2.2. Consequently,

$$\tilde{\boldsymbol{C}}_j^{\boldsymbol{m}} = (\tilde{\boldsymbol{S}}_j^{\boldsymbol{m}})^{-1} \text{ and } \tilde{\boldsymbol{S}}_j^{\boldsymbol{m}} = \boldsymbol{S}_j^{\boldsymbol{0}} + \sum_i \eta_i^m d_i^m \boldsymbol{H}_{\boldsymbol{i,j}}^{\boldsymbol{ve,m}} \tag{7}$$

where d_i^m is the same damage variable as that used during the computation of the long-time stress in the equation (3), $\boldsymbol{S}_j^{\boldsymbol{0}}$ is the initial viscoelastic compliance tensor and $\boldsymbol{H}_{\boldsymbol{i,j}}^{\boldsymbol{ve,m}}$ is the viscoelastic compliance tensor associated with the damage variable d_i^m.

To use the same damage variables is not unreasonable as the matrix damage and the viscoelasticity are both inherent in the resin.

4 Material model formulation for finite strain

Given the shear behaviour of the fabric composite ply with the possibility of large rotations of the yarns, the base (or undeformed) and the current (deformed) configurations cannot be assumed identical. It is therefore essential to extend the model in finite strain.

With few exceptions, Lagrangian (or Material) coordinates are used to describe the deformations of a solid. This approach facilitates the formulation of material constitutive models since the position and the physical properties of solid particles are described according to a reference position of these material particles and time.

4.1 Expression of tensors

As the objective is the formulation of a constitutive model in finite strain for structural analysis, it is appropriate to recall the definition of strain and stress tensors.

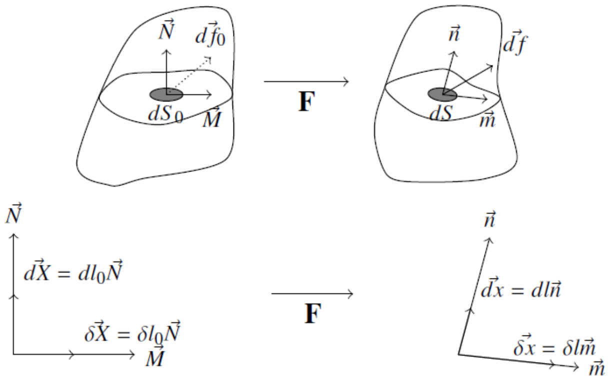

Fig.2: Deformation of a body with representation of undeformed and current configurations.

Let $\vec{dx}$ be a position vector which describes material points in the undeformed configuration. The material points in the deformed configurations are now described by $\vec{dx}$ (Fig.2). The change of the material points is defined by the function $x_i = m_i(X_I, t)$. Hence, the differential is

$$dx_i = \frac{\partial x_i}{\partial X_J} dX_J \tag{8}$$

which leads to the definition of the deformation gradient tensor $\boldsymbol{F}$ by

$$\vec{dx} = \boldsymbol{F}\,\vec{dX} \text{ with } \boldsymbol{F}_{iJ} = \frac{\partial x_i}{\partial X_J}. \tag{9}$$

The transformation of a volume element dv_0 to a volume element dv is given by the relation

$$dv = J \cdot dv_0 \text{ with } J = \det(\boldsymbol{F}). \tag{10}$$

The elongation of a position vector in the $\vec{N}$ direction is defined as

$$\varepsilon(\vec{N}) = \frac{dl - dl_0}{dl_0}. \tag{11}$$

Lastly, the shear angle is determined by

$$\gamma(\vec{M}, \vec{N}) = (\vec{M}; \vec{N}) - (\vec{m}; \vec{n}). \tag{12}$$

From now, a distinction should be made between the tensors expressed according to the base configuration or the current configuration.
First of all, suppose that the reference is the base configuration. The scalar product of the material vectors in the current configuration, evaluated according to these material vectors in the base configuration, is given by

$$\vec{dx} \cdot \vec{\delta x} = \vec{dX} \cdot \boldsymbol{C} \cdot \vec{\delta X} \tag{13}$$

where $\boldsymbol{C} = \boldsymbol{F}^{\mathrm{T}}\boldsymbol{F}$ is the right Cauchy-Green deformation tensor. Similarly, the variation of the scalar product according to the material vectors in the base configuration is defined with

$$\vec{dx} \cdot \vec{\delta x} - \vec{dX} \cdot \vec{\delta X} = 2\,\vec{dX} \cdot \boldsymbol{E} \cdot \vec{\delta X} \tag{14}$$

where $\boldsymbol{E} = 1/2 \cdot (\boldsymbol{C} - \boldsymbol{1})$ is called the Green-Lagrange strain tensor with $\boldsymbol{1}$ the identity tensor.
In a consistent manner, but by considering the current configuration as reference, the strain can be described by the Almansi tensor $\boldsymbol{A} = 1/2 \cdot (\boldsymbol{1} - \boldsymbol{B})$ where $\boldsymbol{B} = \boldsymbol{F}\boldsymbol{F}^{\mathrm{T}}$ is the left Cauchy-Green deformation tensor.
Now that the different strain tensors are defined, the stresses have to be evaluated according to both configurations too.

$\vec{df}$ is a force which acts on the current body and is the only one measurable from the experiments. By expressing the force vector according to the base or the current configuration, this leads to

$$\begin{cases} \vec{df} = \boldsymbol{T}\,\vec{n}\,dS \\ \vec{df} = \boldsymbol{\Pi}\,\vec{N}\,dS_0 \end{cases} \tag{15}$$

with $\boldsymbol{T}$ the Cauchy stress tensor and $\boldsymbol{\Pi}$ the first Piola-Kirchoff stress tensor (PK1). Note that whereas $\boldsymbol{T}$ is symmetric, $\boldsymbol{\Pi}$ because of its mixed nature is not.

Let $\vec{df}_0$ be a virtual force, seen as the equivalent of $\vec{df}$ which may act on the reference configuration. $\vec{df}_0$ has no physical existence and is the transposition of $\vec{df}$ in the base configuration: $\vec{df}_0 = \boldsymbol{F}^{-1}\,\vec{df}$.
Hence, a new stress tensor integrally based on the reference configuration can be defined through the relation $\vec{df}_0 = \boldsymbol{S}\,\vec{N}\,dS_0$ with

$$\boldsymbol{S} = \boldsymbol{F}^{-1}\boldsymbol{\Pi} \tag{16}$$

where $\boldsymbol{S}$ is called the second Piola-Kirchoff stress tensor (PK2) and $\boldsymbol{S}$ is symmetric.
As a result, the behaviour law in finite strain is according to the reference configuration by finding the relation $\boldsymbol{S} = s(\boldsymbol{E})$ which ensures the material objectivity and preserves the material direction during the deformations.
However, LS-Dyna uses an Updated Lagrangian Framework and the stress need to be expressed according to the current configuration (Cauchy stress). The Cauchy stress are recovered from a push forward operation: $\boldsymbol{T} = J^{-1}\boldsymbol{F}\boldsymbol{S}\boldsymbol{F}^{\mathrm{T}}$.
After application of the finite strain framework to the viscoelastic matrix damage model and the appropriate parameter identification, an example of simulation can be seen Fig.3.

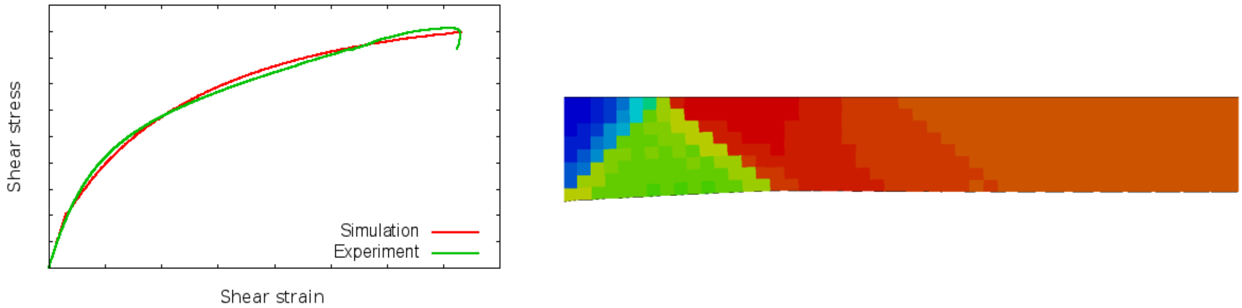

Fig.3: Experimental/numerical comparison of a monotonic tensile shear test (±45°) on a Non-Crimp Fabric coupon and visualisation of the damage parameter (red is maximum) on a quartile of coupon.

5 Recomputation of the strains for layered materials

The shell element formulations in LS-Dyna use First-order Shear Deformation Theory (FSDT) with correction factor. This theory leads to inaccurate transverse shear distribution through the thickness of a layered material. It is therefore impossible to predict eventual interlaminar damages and their effects on the transverse shear thickness of the plate.
Kim and Cho [4] have developed a higher order zigzag theory with interfacial imperfections which requires only five degrees of freedom to take into account the delamination effects. Through least square approximation, Kim et al. [5] have proposed a methodology to ensure the energy equivalence between this formulation and the Mindlin-Reissner Theory (FSDT), called Enhanced First-Order Shear Deformation Theory.
However this methodology is used during the pre-processing phase to compute an effective transverse shear modulus and the associated shear correction factor. This a priori computation does not allow the study of the damage evolution during a finite element simulation. The equilibrium balance and the determination of the displacement field based on the EFSDT has been integrated in the user material subroutine for accurate simulations of impacts on layered plates.

5.1 Displacement theory

Note: The Greek indices take values of 1 or 2, $\delta_{\alpha\beta}$ is the Kronecker symbol and $H(\bullet)$ is the Heaviside function.
The higher-order zigzag displacement field is given by

$$u_\alpha = u_\alpha^0 - u_{3,\alpha}^0 x_3 + \Phi_{\alpha\beta}\phi_\beta + \sum_{d=1}^{D} \bar{u}_\alpha^{(d)} H(x_3 - x_3^{(d)}) \text{ and } u_3 = u_3^0 . \tag{17}$$

The u^0 variables represent the displacements of a point on the reference plane at the position $\Omega \in [-1;1]$ and

$$\Phi_{\alpha\beta} = \left(x_3^3 - \frac{3h^2}{4} x_3 \right) \delta_{\alpha\beta} + \sum_{k=1}^{N-1} a_{\alpha\beta}^{(k)} \left(-\frac{x_3}{2} - \frac{x_3^2}{2h} + (x_3 - x_3^{(k)}) H(x_3 - x_3^{(k)}) \right) \tag{18}$$

with $a^{(k)}$ a coefficient which represents the change of slope at the k-th interface.

$\bar{u}^{(d)}$ is the displacement jump due to the delamination at the interface (d) and is defined by

$$\bar{u}_{\alpha}^{(d)} = R_{\alpha\beta}^{(d)} \sigma_{\beta 3}^{(d)} \tag{19}$$

with $R^{(d)}$ the compliance of the d-th interface.

Thus, the displacement field can be rewritten

$$u_{\alpha} = u_{\alpha}^0 - u_{3,\alpha}^0 x_3 + \Psi_{\alpha\beta} \phi_{\beta} \tag{20}$$

The newly introduced function Ψ is defined by

$$\Psi_{\alpha\beta} = \Phi_{\alpha\beta} + T_{\alpha\beta} + C_{\alpha\beta}^{w} \tag{21}$$

with

$$T_{\alpha\beta} = \sum \left(R_{\alpha\gamma}^{(k)} Q_{\gamma\omega}^{s,(k)} \Phi_{\omega\beta,3}(x_3^{(k)}) \right) H(x_3 - x_3^{(k)}) \tag{22}$$

and

$$C_{\alpha\beta}^{w} = -\frac{1}{h} \int_{-\frac{h}{2}(1+\Omega)}^{\frac{h}{2}(1-\Omega)} (\Phi_{\alpha\beta} + T_{\alpha\beta}) dx_3 \, . \tag{23}$$

C^{w} is added to make the displacements of the reference plane the variables.

5.2 Energy balance

The Mindlin-Reissner plate theory provides a displacement field defined as follows:

$$v_{\alpha} = v_{\alpha}^0 + \theta_{\alpha} x_3 \, , \tag{24}$$

$$v_3 = v_3^0 \, . \tag{25}$$

This displacement field is considered as the average displacement of the EFSDT through the least square approximation and it leads to the following relationship:

$$v_{\alpha}^0 = u_{\alpha}^0 + \int_{-\frac{h}{2}(1+\Omega)}^{\frac{h}{2}(1-\Omega)} (\Psi_{\alpha\beta}) dx_3 \times \phi_{\beta} \, , \tag{26}$$

$$\theta_{\alpha} = -u_{3,\alpha}^0 + \frac{12}{h^3} \int_{-\frac{h}{2}(1+\Omega)}^{\frac{h}{2}(1-\Omega)} (x_3 \Psi_{\alpha\beta}) dx_3 \times \phi_{\beta} \, , \tag{27}$$

$$v_3^0 = u_3^0 \, . \tag{28}$$

Thus, the strain relationship is given by

$$\varepsilon^0 = \gamma^0 - \tilde{\boldsymbol{C}} \kappa_h \, , \tag{29}$$

$$\kappa = \lambda - \tilde{\boldsymbol{\Gamma}} \kappa_h \, , \tag{30}$$

$$\varepsilon_s^0 = \tilde{\boldsymbol{\Gamma}}_s^{-1} \gamma_s^0 . \tag{31}$$

The tensors $\tilde{\boldsymbol{C}}$, $\tilde{\boldsymbol{\Gamma}}$ and $\tilde{\boldsymbol{\Gamma}}_s$ are obtained thanks to an iterative loop which minimizes the difference between the internal energy of the FSDT and of the EFSDT [6].
The displacement field is then recovered by following the equations

$$u_\alpha^0 = v_\alpha^0 - v_{3,\alpha}^0 + \left(\Psi_{\alpha\beta} - \tilde{C}_{\alpha\beta}\right)\tilde{\Gamma}_{\beta\omega}^{-1}\left(\theta_\omega + v_{3,\omega}^0\right) \tag{32}$$

and

$$u_3 = v_3^0 . \tag{33}$$

5.3 Principle

The classical scheme in LS-Dyna, and by using **`ihyper≠0`** in **`*MAT_USER_DEFINED_MATERIAL`**, provides the deformation gradient as input of the user material subroutine. From that, the Cauchy stress has to be returned (Fig.4).

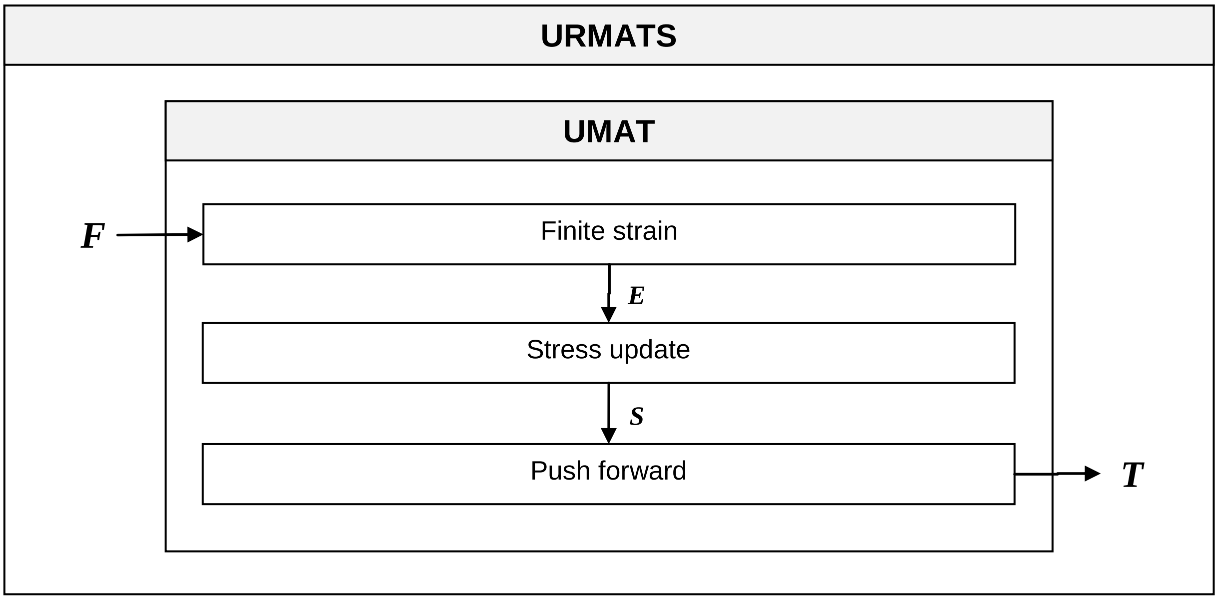

Fig.4: Classical Total Lagrangian UMAT subroutine formulation.

In the case of the present theory, the deformation gradient of the reference plane and the gradient of the deformation gradient of the reference plane are needed as input of the umat subroutine. Thanks to a light modification of the **`urmats`** subroutine (in the dyn21.f file) and by using the **`usrshl_defgrad`** function, these data are passed as input parameters.
For each element and for each new time step the energy balance is achieved. The interlaminar damage is then computed thanks to the stress computation at the interface and these data are stored to be used by the umat subroutine for the next integration points of the element and for the future time steps.
From this, the deformation gradient at the current integration point is computed and the classical algorithm can be recovered (Fig.5).
An example of transverse shear stress distribution computed by the present theory is provided Fig.6.

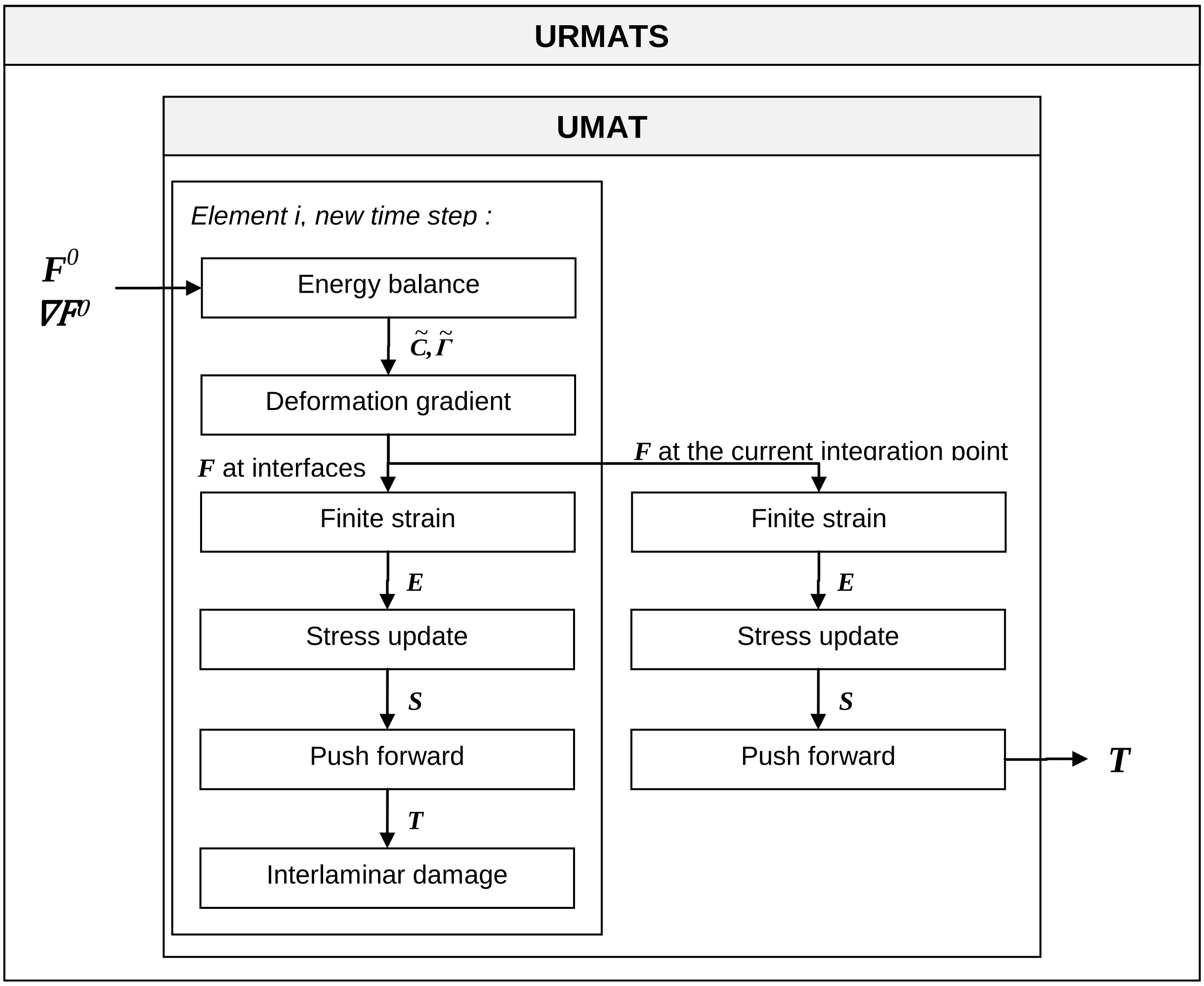

Fig.5: Total Lagrangian UMAT subroutine formulation for layered materials.

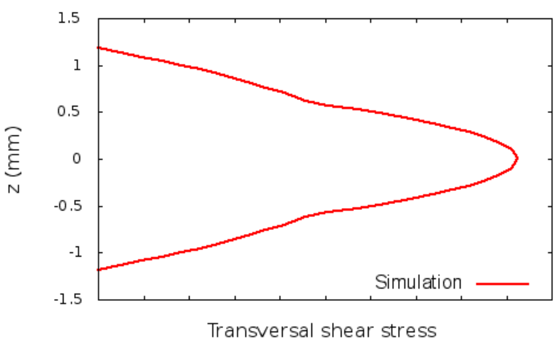

Fig.6: Transverse shear stress of a layered [45°,0°]s unbalanced textile composite during 3-point bending obtained by application of the EFSDT in the user material model.

6 Summary

The high specific stiffness and strength, the ease of shaping as well as the great impact performance of layered fabric reinforced polymers encourage their diffusion in the automotive industry. In order to increase the predictability of explicit finite element analysis (FEA) a material model intended for impact simulations has been developed.
The material model combines the intralaminar behaviour and the effect of delamination without using computationally expensive methods, such as the use of cohesive elements. This allows the use of one element across the thickness of the laminate and an interaction between intra- and interlaminar damage.

The intralaminar behaviour model is based on the explicit formulation of the matrix damage model developed by the ONERA. Coupling with a Maxwell-Wiechert model, the viscoelasticity is included without losing the direct explicit formulation. The numerical instabilities, due to the strain-softening from the fibre failure, are managed by the smeared-crack approach depending on the element size. Additionally, the intralaminar model is formulated under a total Lagrangian scheme in order to maintain consistency for finite strain by tracking the material direction and by ensuring objectivity.
Thanks to the membrane deformation provided by a Mindlin-Reissner shell formulation and the stacking sequence, the material model is able to compute the through-thickness strains. Based on a higher-order zigzag displacement theory with interfacial imperfections, the strains are established by using an internal loop, which ensures the internal energy equilibrium between both plate theories. The stress at Gauss points are then computed by using the intralaminar model.
This work provides a way to precisely predict the damage and the behaviour of composite plates under impact loading.

7 Acknowledgment

The present research work has been supported by International Campus on Safety and Intermodality in Transportation, the Region Nord Pas de Calais, the European Community, the Délégation Régionale à la Recherche et à la Technologie, the Ministère de l’Enseignement Supérieur et de la Recherche, the Centre National de la Recherche Scientifique and TOYOTA MOTOR EUROPE: the authors gratefully acknowledge the support of these institutions.

8 Literature

[1] MARCIN, Lionel. Modélisation du comportement, de l'endommagement et de la rupture de matériaux composites à renforts tissés pour le dimensionnement robuste de structures. 2010. Thèse de doctorat. Université Bordeaux 1.
[2] CHABOCHE, J.-L. et MAIRE, J.-F. A new micromechanics based CDM model and its application to CMC's. Aerospace Science and Technology, 2002, vol. 6, no 2, p. 131-145.
[3] KALISKE, M. A formulation of elasticity and viscoelasticity for fibre reinforced material at small and finite strains. *Computer Methods in Applied Mechanics and Engineering*, 2000, vol. 185, no 2, p. 225-243.
[4] KIM, Jun-Sik et CHO, Maenghyo. Efficient higher-order shell theory for laminated composites with multiple delaminations. *AIAA journal*, 2003, vol. 41, no 5, p. 941-950.
[5] KIM, Jun-Sik, OH, Jinho, et CHO, Maenghyo. Efficient analysis of laminated composite and sandwich plates with interfacial imperfections. *Composites Part B: Engineering*, 2011, vol. 42, no 5, p. 1066-1075.
[6] KIM, Jun-Sik et CHO, Maenghyo. Enhanced modeling of laminated and sandwich plates via strain energy transformation. *Composites Science and Technology*, 2006, vol. 66, no 11, p. 1575-1587.