New Features in LS-DYNA EFG Method for Solids and Structures Analysis

C. T. Wu

LSTC

Summary:

In this presentation, an update on LS-DYAN EFG method for solids and structures analysis will be given. Several features were developed in the past two years to solve specific challenging problems as well as to improve the efficiency. This talk will emphasize on three new features including an adaptive Meshfree scheme based on a local Maximum Entropy approximation for metal forging and extrusion analysis, a semi-Lagrangain formulation in foam materials under severe compression, and a discrete meshfree approach in the failure analysis of brittle materials. Several practical examples are included to demonstrate these capabilities.

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New Features in LS-DYNA EFG Method for Solids and Structures Analysis

C. T. Wu*, Yong Guo, Jing Xiao Xu and Hong Sheng Lu
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May 14th ~ 15th, 2009
Salzburg, Austria

7th European LS-DYNA Conference

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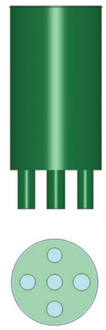
Current EFG Formulations for Solids and Structures Analysis

- **Metal materials in Forging/Extrusion analysis: Adaptive formulation**
- **Foam materials: Semi-Lagrangian kernel formulation** } **Stabilized Method**
- **Rubber materials: Lagrangian kernel formulation** } **Stabilized Method**
- **Quasibrittle material fracture: Strong discontinuities formulation**
- **E.O.S. materials: Eulerian kernel formulation (trial version)**
- **Meshfree Shell: Lagrangian kernel, adaptivity …**

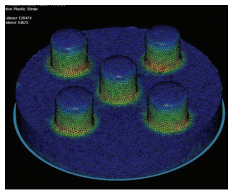
Adaptive Methods for Manufacturing Simulations



Reasons for Adaptivity

- High accuracy requirement (surface representation, high gradient …)
- Residual stress effects the crash result

Current Numerical Limitations



- RH-adaptivity for solids (H-adaptivity is limited to shell structures).
- No failure is allowed if failure energy is important (can not be extended to metal cutting, riveting ..)
- Do not apply to rubber-like materials

1. Adaptive EFG Method

Adaptive Forging/Extrusion analysis

- An explicit/implicit solver coupled with thermal analysis.
- Introduce a fast transformation meshfree method and a modified Maximum Entropy approximation to improve the efficiency.
- A second-order interpolation scheme for state variable transfer.
- Include global/local adaptive refinements.
- Available in SMP and MPP.

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EFG Fast Transformation Method

- Momentum equation

$$\rho \dot{\boldsymbol{v}} = \nabla_x \cdot \boldsymbol{\sigma} + \boldsymbol{b}$$

- Continuity equation

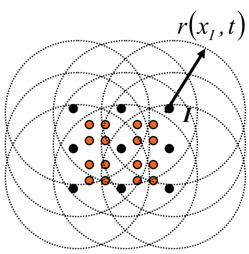
$$\dot{\rho} = -\rho \nabla_x \boldsymbol{v}$$

$$\int_\Omega \delta \boldsymbol{v} \cdot \rho \dot{\boldsymbol{v}} d\Omega = -\int_\Omega \nabla \delta v : \boldsymbol{\sigma} \, d\Omega + \int_\Omega \delta \boldsymbol{v} \cdot \boldsymbol{b} d\Omega + \int_\Gamma \delta \boldsymbol{v} \cdot \boldsymbol{\tau} d\Gamma$$

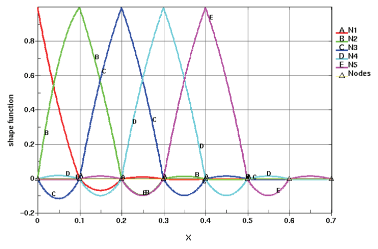
$$m_I \dot{\boldsymbol{v}}_I = -\sum_I \nabla_x \Phi_I(\boldsymbol{x}_s) \cdot \boldsymbol{\sigma}_s V_s$$

$$\dot{\rho}_s = -\rho_s \sum_I \boldsymbol{v}_I \cdot \nabla_x \Phi_I(\boldsymbol{x}_s)$$

$$u_{\Omega}^h(\boldsymbol{x}) = \sum_{\substack{I \in \Omega \\ x \in \Omega}} \Phi_I^{[n]}(\boldsymbol{x}) \cdot \sum_{J \in \Omega} \hat{\Psi}_J^{[m]}(\boldsymbol{x}_I) \boldsymbol{x}_J \equiv \sum_{\substack{I \in \Omega \\ x \in \Omega}} \bar{\Psi}_I^{[m]}(\boldsymbol{x}) \boldsymbol{x}_I$$



● Particle
● Stress point



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EFG Modified Maximum Entropy Method

Local MAXENT (Ortiz and Arroyo, 2006)

$$(MAXENT) \quad \text{maximize} \quad H(\boldsymbol{p}) = \beta(\boldsymbol{x}) \sum_{i=1}^{N} p_i \left| \boldsymbol{x}_i - \boldsymbol{x} \right|^2 + \sum_{i=1}^{N} p_i \log p_i$$

$$\text{subject to} \quad p_i \geq 0, i = 1, \ldots, N$$

$$\sum_{i=1}^{N} p_i = 1$$

$$\sum_{i=1}^{N} p_i (\boldsymbol{x}_i - \boldsymbol{x}) = \boldsymbol{0}$$

— for $\beta \in [0, +\infty)$, *H(**p**) is continuous and strictly convex in solution* (well-behaved mass matrix, monotonicity, variation diminishing …)

— less dependent

— difficult to decide β

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EFG Modified Maximum Entropy Method

Define the partition function Z: $Z(\boldsymbol{x},\boldsymbol{\lambda}) \equiv \sum_{i=1}^{N} \phi_i(\boldsymbol{x}) e^{\lambda \cdot (x - x_i)/r_i}$

where $\phi_i(\boldsymbol{x})$ is the kernel function at node i

$\boldsymbol{r}_i$ is the support size of kernel at node i

- Non-negative approximation
- Smoothness in irregular nodes
- Less dependence
- Kronecker-Delta at boundary

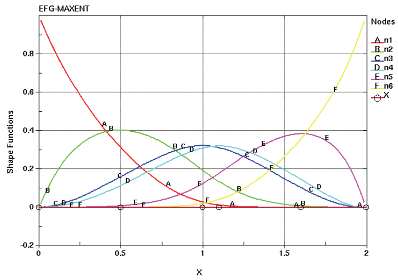
The unique solution of MAXENT is proven to be

$$p_i(\boldsymbol{x},\boldsymbol{\lambda}) = \frac{\phi_i(\boldsymbol{x}) e^{f_i(x,\lambda)}}{Z(\boldsymbol{x},\boldsymbol{\lambda})} \quad \forall p_i \geq 0, i = 1,\ldots,N$$

satisfying $\sum_{i=1}^{N} p_i = 1$

$$\sum_{i=1}^{N} p_i\,(\boldsymbol{x}_i - \boldsymbol{x}) = \boldsymbol{0}$$

where $f_i(\boldsymbol{x},\boldsymbol{\lambda}) = \boldsymbol{\lambda} \cdot [(\boldsymbol{x} - \boldsymbol{x}_i)/\boldsymbol{r}_i]$



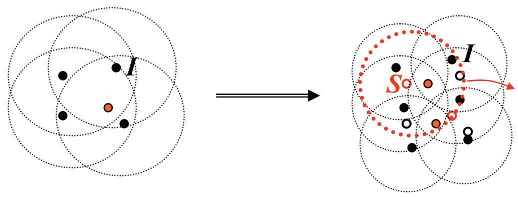
— implicit solve; 3~5 iterations

Mesh-free Interpolation for Data Transfer in Adaptivity

Current variable update : $f_s^{n+1} \approx A_{\alpha s}^{n+1} \tilde{f}_{\alpha} = A_{\alpha s}^{n+1} A_{\alpha\beta}^{n^{-1}} f_{\beta}^{-}$

$$A_{IJ} = \overline{\Phi}_I^{[m]}(x_J)$$

$\overline{\Psi}_I^{[m]}(x^-, t_{n+1})$ $\overline{\Psi}_I^{[m]}(x^+, t_{n+1})$



$\overline{\Phi}_S^{[m]}(x^+, t_{n+1})$

- Particle
- Stress point

- Old Particle
- Old Stress point
- New Particle
- New Stress point

Input Format
*SECTION_SOLID_EFG

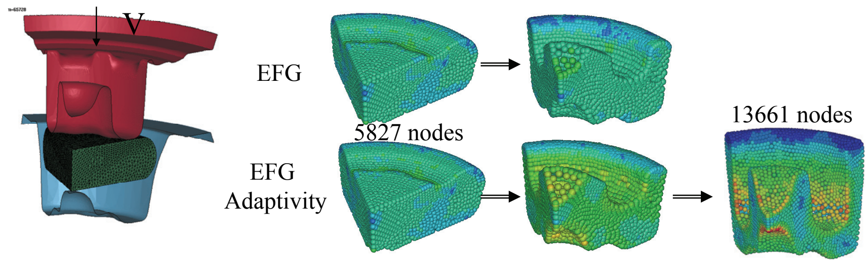
Essential Boundary Conditions

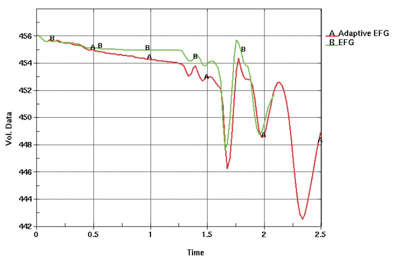
Card 2

Variable	DX	DY	DZ	ISPLINE	IDILA	IEBT	IDIM	TOLDEF
Type	F	F	F	I	I	I	I	F
Default	1.01	1.01	1.01	0	0	1	1	0.01

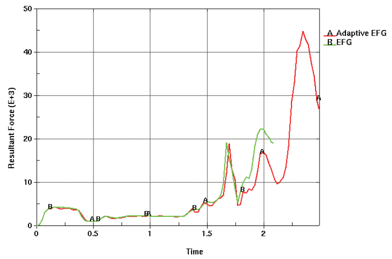
IEBT EQ. 1: Full transformation (default)
EQ.–1: (w/o transformation)
EQ. 2: Mixed transformation
EQ. 3: Coupled FEM/EFG
EQ. 4: Fast transformation
EQ.–4: (w/o transformation)
EQ. 5: Fluid particle (trial version)
EQ. 7: Modified Maximum Entropy approximation

Forging Simulation





Volume change (1.7%)



Force

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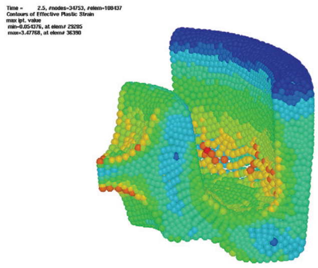
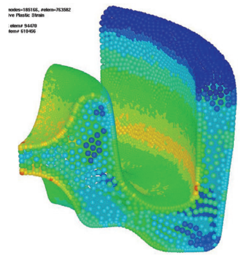


Global Refinement



Local Refinement

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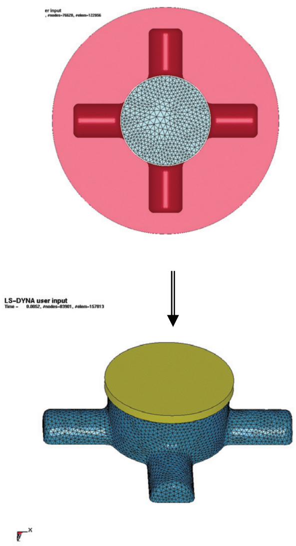
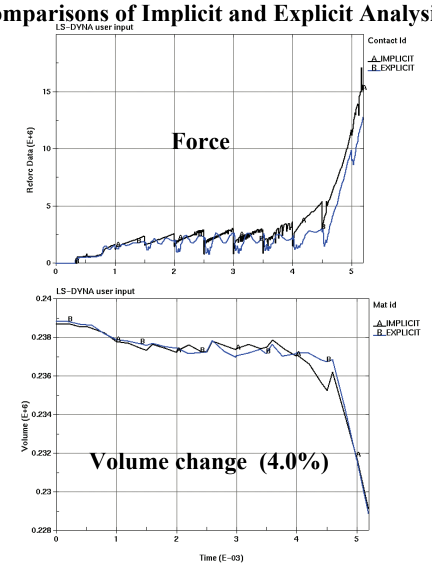
Extrusion Simulation



Comparisons of Implicit and Explicit Analysis



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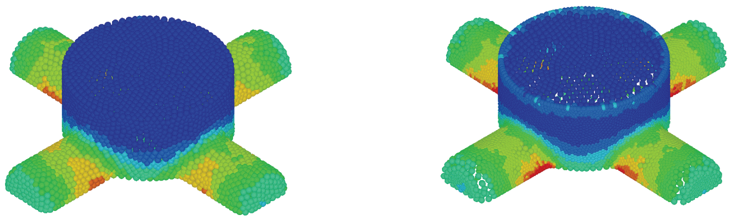

Global Refinement Local Refinement

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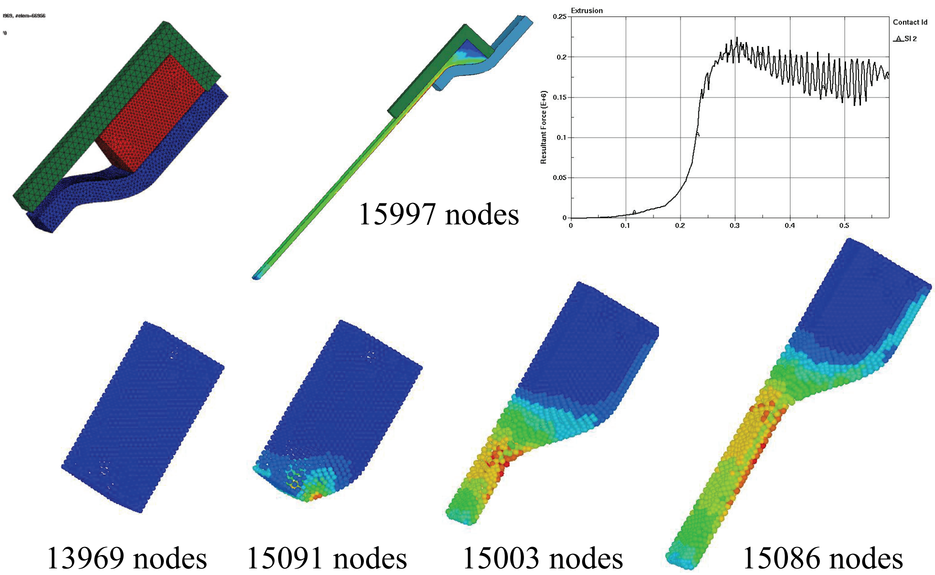
Extrusion with Thermal Coupling



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2. The Stabilized EFG Method with Kernel Switch

The Stabilized EFG Method with kernel switch

- Is a one-point integration scheme + gradient type hourglass control.
- Assumed strain method for nearly incompressible materials.
- Designed especially for foam and rubber materials.
- The speed is between FEM reduced integration element (#1) and full integration element (#2)
- A switch to full integration (rubber) or Semi-Lagrangian kernel (foam) is allowed in large deformation range.
- Available in SMP explicit and MPP explicit.

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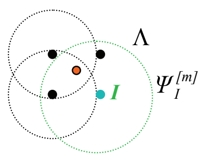
Gradient Type Stabilized EFG Method

$$\Psi_I^{[m]} = \Psi_{I0}^{[m]} + (x - x_0)\Psi_{I,x}^{[m]} + (y - y_0)\Psi_{I,y}^{[m]} + (z - z_0)\Psi_{I,z}^{[m]} + O_2$$

$$\tilde{\boldsymbol{B}}_I^{[m]} = \boldsymbol{B}_{I0}^{[m]} + (x - x_0)\boldsymbol{B}_{I,x}^{[m]} + (y - y_0)\boldsymbol{B}_{I,y}^{[m]} + (z - z_0)\boldsymbol{B}_{I,z}^{[m]}$$



Assumed Strain Method

$$\tilde{\boldsymbol{\varepsilon}} = \bar{\boldsymbol{B}}\dot{\boldsymbol{U}}$$

$$\bar{\boldsymbol{B}} = \boldsymbol{B}_0 + \underbrace{\bar{\boldsymbol{B}}_x(x - x_0) + \bar{\boldsymbol{B}}_y(y - y_0) + \bar{\boldsymbol{B}}_z(z - z_0)}_{anti\text{-}hourglass}$$

$$\bar{\boldsymbol{B}}_x = \begin{bmatrix} \frac{\partial^2}{\partial x^2} & -\frac{\partial^2}{2\partial x\partial y} & -\frac{\partial^2}{2\partial x\partial z} \\ -\frac{\partial^2}{2\partial x^2} & \frac{\partial^2}{\partial x\partial y} & -\frac{\partial^2}{2\partial x\partial z} \\ -\frac{\partial^2}{2\partial x^2} & -\frac{\partial^2}{2\partial x\partial y} & \frac{\partial^2}{\partial x\partial z} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Psi^T & & \\ & \Psi^T & \\ & & \Psi^T \end{bmatrix};$$

$$\left.\begin{array}{l} \sum_{I\in\Lambda}^{n} \Psi_{I,x}^{[m]}(x) = 0 \\ \sum_{I\in\Lambda}^{n} \Psi_{I,x}^{[m]}(x)\cdot x_I = 1, \\ \sum_{I\in\Lambda}^{n} \Psi_{I,x}^{[m]}(x)\cdot y_I = 0 \end{array}\right. \quad \overset{m \geq 1}{\Lambda = \{I | 1,2,\cdots NP\}} \Rightarrow \underbrace{\bar{\boldsymbol{B}}_x \bullet \boldsymbol{1} = \boldsymbol{0}}_{parition\ of\ nullity}$$

Total Lagrangian

$$\sigma_{ij} \approx F_{ik}S_{kj}\big|_{(x_0,y_0,z_0)} + F_{ik,x}S_{kj}\big|_{(x_0,y_0,z_0)}(x - x_0) + F_{ik,y}S_{kj}\big|_{(x_0,y_0,z_0)}(y - y_0) + F_{ik,z}S_{kj}\big|_{(x_0,y_0,z_0)}(z - z_0)$$

Semi-Lagrangian Kernel in Foam Material

Convective velocity ***C*** due to Semi-Lagrangian or Eulerian kernel

$$\boldsymbol{v}^{-} = \frac{\partial \boldsymbol{x}\ (\boldsymbol{X},t_{n+1})}{\partial t}\Big|_{X}\ (material\ time\ frame) \qquad \boldsymbol{C} = \boldsymbol{v}^{-} - \boldsymbol{v}^{+}$$

$$\boldsymbol{v}^{+} = \frac{\partial \boldsymbol{x}\ (\boldsymbol{\chi},t_{n+1})}{\partial t}\Big|_{\chi}\ (reference\ time\ frame) \qquad \dot{\boldsymbol{f}} = \frac{\partial \boldsymbol{f}}{\partial t}\Big|_{\chi} + (\boldsymbol{C}\cdot\nabla)\boldsymbol{f}$$

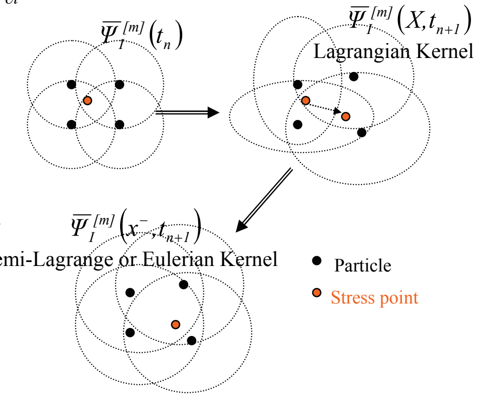
- ❑ Lagrnagian phase : $\overline{\Psi}_J^{[m]}(\boldsymbol{x},t_n) = \overline{\Psi}_J^{-[m]}$
- ❑ Transport phase : $\frac{\partial \boldsymbol{f}}{\partial t}\Big|_{\chi} + (\boldsymbol{C}\cdot\nabla)\boldsymbol{f} = 0$

$$\nabla \boldsymbol{f} = \sum_J \frac{\partial \overline{\Psi}_J^{[m]}}{\partial \boldsymbol{x}} \tilde{\boldsymbol{f}}_I;\ \boldsymbol{f}_I = \sum_J \overline{\Psi}_J^{[m]}(\boldsymbol{x}_I,t_n)\tilde{\boldsymbol{f}}_J$$

$$\longrightarrow \overline{\Psi}_J^{[m]}(\boldsymbol{x}^{+}) = \overline{\Psi}_J^{[m]}(\boldsymbol{x}^{-}) - (\boldsymbol{x}^{+} - \boldsymbol{x}^{-})\cdot \frac{\partial \overline{\Psi}_J^{[m]}}{\partial \boldsymbol{x}}\Big|_{x^{-}} + \cdots$$

$$\boldsymbol{f}^{+} \approx \sum_J \overline{\Psi}_J^{[m]}(\boldsymbol{x}^{+})\tilde{\boldsymbol{f}}_J$$

Stress recovery scheme is conservative, consistent and monotonic !



Input Format
*SECTION_SOLID_EFG

Domain Integration Schemes

Card 2

Variable	DX	DY	DZ	ISPLINE	IDILA	IEBT	IDIM	TOLDEF
Type	F	F	F	I	I	I	I	F
Default	1.01	1.01	1.01	0	0	1	1	0.01

IEBT EQ. 1: Full transformation (default)
EQ.–1: (w/o transformation)
EQ. 2: Mixed transformation
EQ. 3: Coupled FEM/EFG
EQ. 4: Fast transformation
EQ.–4: (w/o transformation)
EQ. 5: Fluid particle (trial version)
EQ. 7: Modified Maximum Entropy approximation

IDIM EQ. 1: Local boundary condition method (default)
EQ. 2: Two-points Guass integration
EQ.-1: Stabilized EFG method

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Input Format
*SECTION_SOLID_EFG

Deformation tolerance for the activation of Semi-Lagrangian kernel

Card 2

Variable	DX	DY	DZ	ISPLINE	IDILA	IEBT	IDIM	TOLDEF
Type	F	F	F	I	I	I	I	F
Default	1.01	1.01	1.01	0	0	1	1	0.01

TOLDEF $|TODELF| < 1.0$
= 0.0 : Lagrangian kernel
> 0.0 : Semi-Lagrangian kernel
< 0.0 : Eulerian kernel

Time control for the activation of Semi-Lagrangian kernel or Eulerian kernel

Card 3

Variable	IGL	STIME	IKEN				
Type	I	100.0	I				
Default	0	1.e+20	0				

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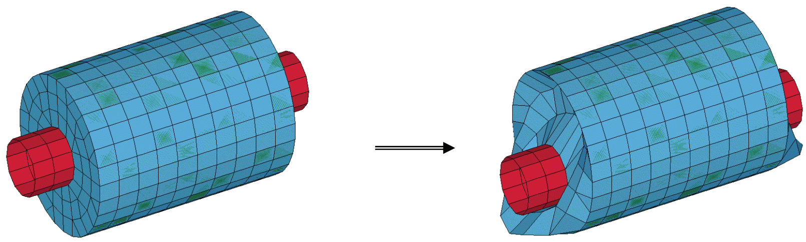
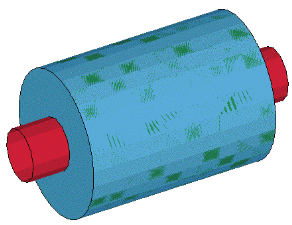
Rubber Bushing Analysis using Stabilized EFG Method





Mooney-Rivlin Rubber
Poisson's =0.4995
Stabilized EFG explicit analysis
Switched to full integration at t=100
Completion at t=150

CPU comparison at t=50

Methods	S-FEM(#1)	F-FEM(#2)	EFG	S-EFG
CPU	1.0	4.1	5.4~12.9	2.6

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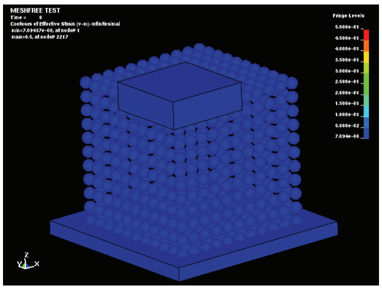
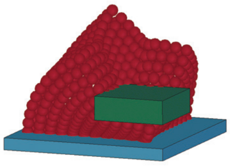
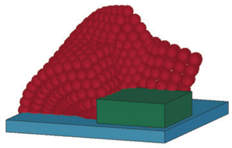
Foam Compression using Stabilized EFG Method and Semi-Lagrangian Kernel



Low Density Foam
Stabilized EFG explicit analysis
Switched to Semi-Lagrangian (TOLDEF=0.01)



Original EFG



EFG +
Semi-Lagrangian Kernel

Comparison of final deformation

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3. EFG Failure Analysis

Meshfree Failure Analysis

- Is a discrete approach.
- Crack initiation and propagation are governed by cohesive law.
- Crack currently is cell-by-cell propagation and is defined by visibility.
- Minimized mesh sensitivity and orientation effects.
- Applied to quasibrittle materials.

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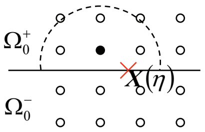
Discrete Cracks

Crack in Meshfree: Visibility Criterion (Belytschko et al.1996)

Intrinsic (Implicit crack) : no additional unknowns



$$x(\eta)=\sum_{I=1}^{2}\Phi_I^{FEM}(\eta)X_I+\frac{1}{2}\left(\sum_{J\in\Omega_0^+}\Psi_J(X(\eta))u_J+\sum_{J\in\Omega_0^-}\Psi_J(X(\eta))u_J\right)$$

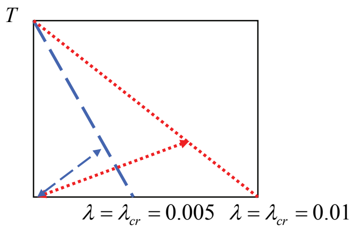
$$\frac{\partial x(\eta)}{\partial\eta}=\sum_{I=1}^{2}X_I\otimes\frac{\partial\Phi_I^{FEM}(\eta)}{\partial\eta}+\frac{1}{2}\left(\sum_{J\in\Omega_0^+}u_J\otimes\frac{\partial\Psi_J(X)}{\partial X}+\sum_{J\in\Omega_0^-}u_J\otimes\frac{\partial\Psi_J(X)}{\partial X}\right)\frac{\partial X(\eta)}{\partial\eta}$$

Initially-rigid Cohesive Law: Redefined Displacement Jump (Sam, Papoulia and Vavasis 2005)

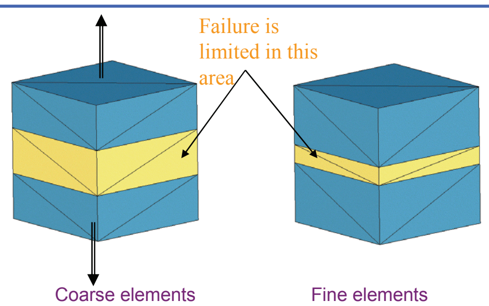
$$\lambda=\sqrt{\left(u_n/(\delta_{0n}+\delta_n)\right)^2+\beta^2\left(u_t/(\delta_{0t}+\delta_t)\right)^2}$$

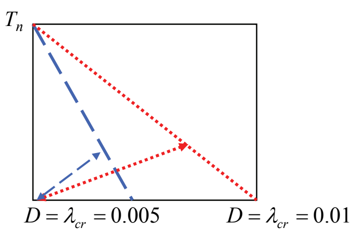
$$T_{efs}\equiv\sqrt{T_n^2+\left(\frac{\beta}{\alpha}\right)^2T_t^2}=T_{max}$$

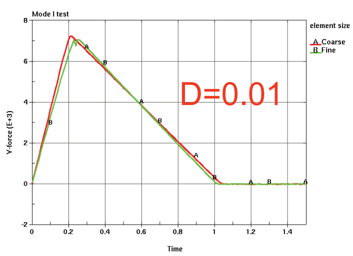
$$T_n=\frac{1-\lambda}{\lambda}\frac{u_n}{\delta_n}\frac{T_{max}}{1-\lambda_{cr}}\quad \textit{and}\quad T_t=\frac{1-\lambda}{\lambda}\frac{u_t}{\delta_t}\frac{\alpha T_{max}}{1-\lambda_{cr}}$$

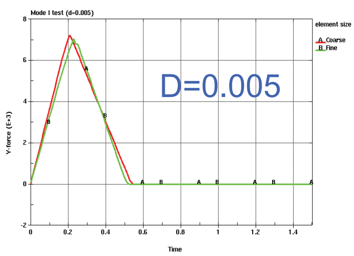


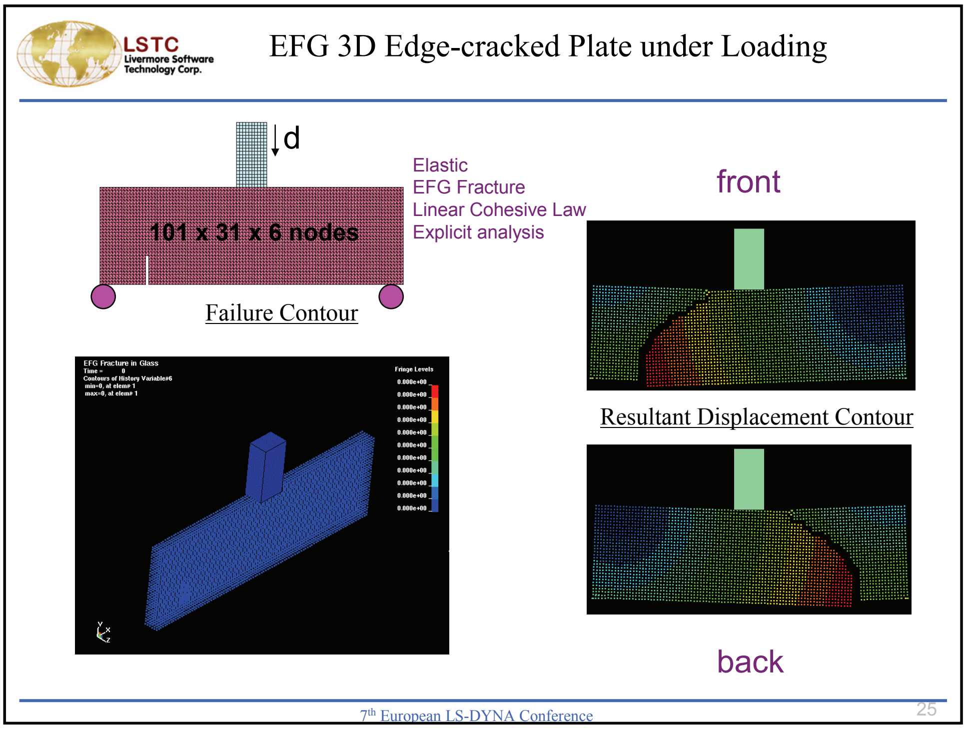
Minimization of Mesh Size Effect in Mode-I Failure Test







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EFG 3D Edge-cracked Plate under Loading
d
101 x 31 x 6 nodes
Elastic
EFG Fracture
Linear Cohesive Law
Explicit analysis
front
Failure Contour
Resultant Displacement Contour
back
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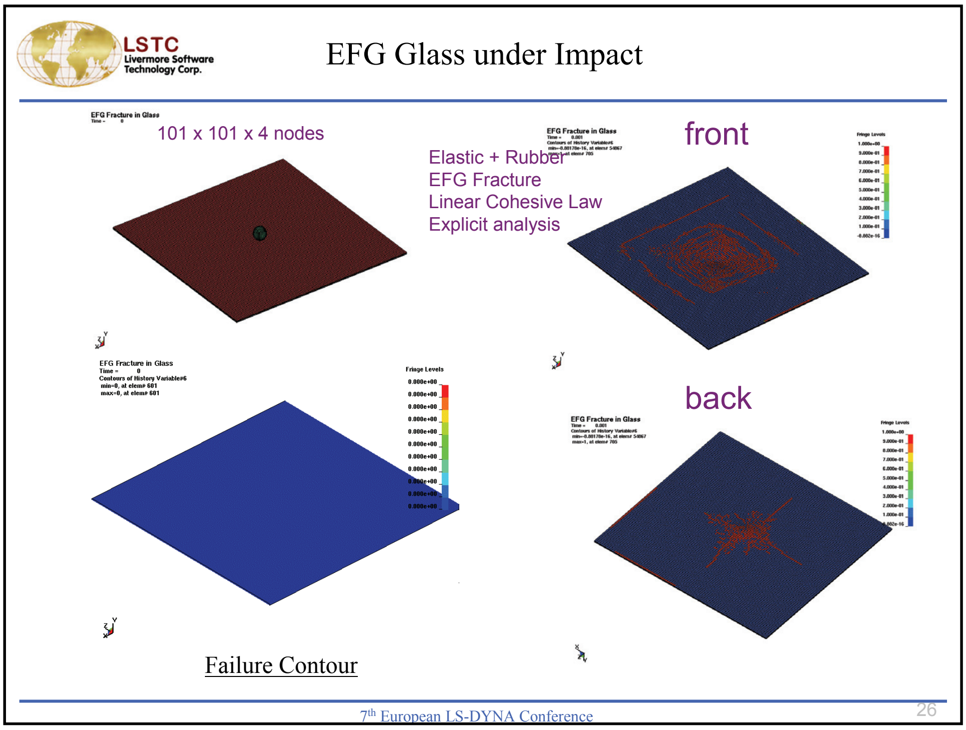
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EFG Glass under Impact
101 x 101 x 4 nodes
Elastic + Rubber
EFG Fracture
Linear Cohesive Law
Explicit analysis
front
back
Failure Contour
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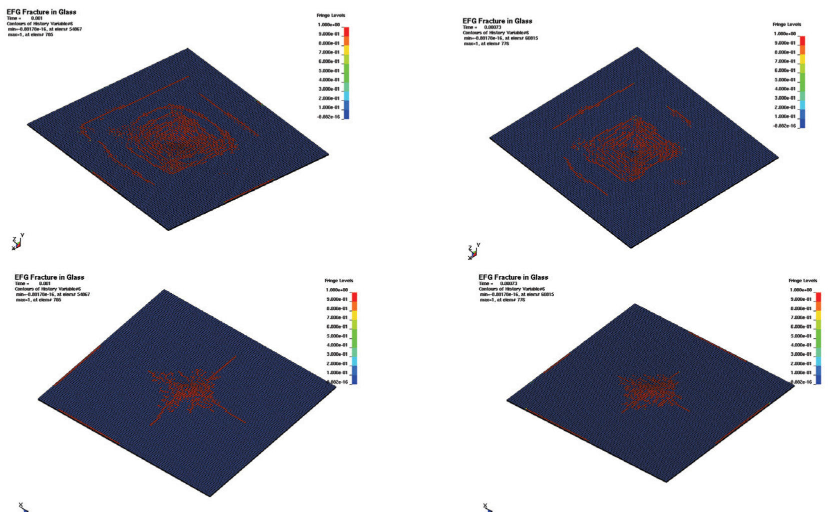

Rigid ball Metal ball

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Conclusions

- Adaptive method is attractive in metal forging/extrusion simulation.
- Stabilized method designed for foam and rubber materials can be used to improve the efficiency in explicit analysis.
- Failure analysis using cohesive model and visibility can be applied to brittle and semi-brittle materials.
- Strong discontinuities formulation including XFEM will be our next focus for a more general failure analysis.