Optional Strain-Rate Forms for the Johnson Cook Constitutive Model and the Role of the Parameter Epsilon_0[1]

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Summary:

A brief review of the standard Johnson-Cook model is presented. Three optional strain-rate forms are introduced and calibrated to laboratory data for A36 steel. Next a brief description of the LS-DYNA implementation of the new strain-rate forms within the existing viscoplastic formulation of the Johnson-Cook model is presented. Finally, all four calibrated strain-rate forms are exercised in single element uniaxial stress test simulations, and the results are compared with the A36 steel effective stress versus effective plastic strain data at three different strain rates. The comparison of the calibrated model response to the quasi-static A36 steel data is used to illustrate the role of the Johnson-Cook parameter $\dot{\varepsilon}_0$.

Keywords:

Johnson-Cook model, Strain rates, visco-plasticity.

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1 Introduction

Under the Momentum Energy Transfer Study (METS) project, the Naval Explosive Ordnance Disposal Technology Division (NAVEODTECHDIV) acquired materials characterization data for ASTM A36 hot rolled steel, including a preliminary set of parameters for the Johnson-Cook constitutive model (Johnson and Cook, 1983 & 1985). As part of the preliminary parameter identification, three alternative forms for the strain-rate portion of the Johnson-Cook model were considered. The present work briefly describes these three strain-rate forms and their implementation in the Johnson-Cook constitutive model available in the general purpose non-linear finite element code LS-DYNA.

Calibration of constitutive model parameters is most often accomplished via regression techniques applied to the laboratory data, without regard for the numerical algorithm used in implementing the constitutive model, e.g. plasticity algorithm. This does not remove the responsibility of the constitutive model end-user to verify that the selected parameters replicate the data from which they were obtained, i.e. reproduce the test data using simple numerical simulations that exercise the constitutive model.

While verifying the implementation of the optional strain-rate forms described in the following sections, an inconsistency between the calibrated model parameters and the algorithmic response was noted. The inconsistency centers on the use of other than the quasi-static stress-strain response to calibrate the basic Johnson-Cook yield and hardening parameters. Closely related to this central focus, is the misunderstanding[2] of the strain-rate normalization parameter $\dot{\varepsilon}_0$ and its role in the calibration of the Johnson-Cook model parameters.

In the following section a brief review of the standard Johnson-Cook model is presented. The three optional strain-rate forms are introduced and calibrated to laboratory data for A36 steel. Next a brief description of the LS-DYNA implementation of the new strain-rate forms within the existing viscoplastic formulation of the Johnson-Cook model is presented. Finally, all four calibrated strain-rate forms are exercised in single element uniaxial stress test simulations, and the results are compared with the A36 steel effective stress versus effective plastic strain data at three widely different strain rates. It is in this comparison of the calibrated model response to the quasi-static A36 steel data where the above mentioned inconsistency is illustrated. The results from a parallel model calibration, using the quasi-static data, are shown to provide a consistent set of results, and illustrate the role of the Johnson-Cook parameter $\dot{\varepsilon}_0$.

2 Johnson-Cook Constitutive Model

The Johnson-Cook constitutive model (1983) is a phenomenological model, i.e. it is not based on traditional plasticity theory, that reproduces several important material responses observed in impact and penetration of metals. The three key material responses are strain hardening, strain-rate effects, and thermal softening. These three effects are combined, in a multiplicative manner, in the Johnson-Cook constitutive model:

[2] Previous versions of the LS-DYNA User Manual description of the Johnson-Cook model parameter $\dot{\varepsilon}_0$ as a time units normalization factor, is an illustration of the misunderstanding associated with this parameter.

$$\sigma_Y = \left[A + B\left(\varepsilon_{eff}^p\right)^N \right]\left(1 + C\ln\dot{\varepsilon}\right)\left[1 - \left(T_H\right)^M\right]$$

$$\varepsilon_{eff}^p = \text{effective plastic strain}$$

$$\dot{\varepsilon} = \frac{\dot{\varepsilon}_{eff}^p}{\dot{\varepsilon}_0} \text{ where } \dot{\varepsilon}_0 \text{ is strain rate used to determine } A, B \text{ \& } N$$

$$T_H = \frac{T - T_R}{T_M - T_R} \text{ Homologous Temperature}$$

$$\mathrm{T_M} = \text{ melt temperature}$$

$$\mathrm{T_R} = \text{reference temperature when determining } A, B \text{ \& } N$$

$$\Delta T = \frac{1}{\rho C_p}\int \sigma d\varepsilon_{eff}^p \text{ where } \rho \text{ is mass density and } C_p \text{ is specific heat}$$

The above yield strength portion of the Johnson-Cook constitutive model has five parameters: A, B, N, C, and M , and three material characteristics: ρ, C_P, and T_M . Additionally, the elastic parameters are required. Typically the shear modulus is input along with an Equation-of-State (EOS) used to define pressure versus volume strain response; for low pressures, the EOS is assumed to be defined by the elastic bulk modulus.

Johnson and Cook (1985) expanded on their basic model with the inclusion of a model for fracture based on cumulative-damage; the LS-DYNA implementation of the Johnson-Cook constitutive model includes this additional model feature. The cumulative-damage fracture model:

$$\varepsilon^F = \left(D_1 + D_2 \exp\left[D_3 \frac{P}{\sigma_{eff}} \right] \right)\left(1 + D_4 \ln\dot{\varepsilon}\right)\left(1 + D_5 T_H\right)$$

$$D = \sum \frac{\Delta\varepsilon_{eff}^p}{\varepsilon^F} \text{ failure occurs when } D = 1$$

$$\sigma_{eff} = \text{ effective stress}$$

$$P = \text{ mean stress (pressure)}$$

is similar in form to the yield strength model with three terms combined in a multiplicative manner to include the effects of stress triaxiality, strain rate, and local heating, respectively. This portion of the Johnson-Cook constitutive model requires an additional five material model parameters.

The Johnson & Cook references (1983, 1985) describe the material characterization tests needed to calibrate the model parameters. The A36 hot rolled steel Johnson-Cook parameters, provided by NAVEODTECHDIV, are provided in Table 1.

Table 1 A36 steel parameters for Johnson-Cook model with standard strain-rate form.

A (ksi)	B (ksi)	N	C	$\dot{\varepsilon}_0$ $\left(\sec^{-1}\right)$	M
41.50	72.54	0.228	0.017	1.0	0.917

3 Strain-Rate Forms

The standard Johnson-Cook model is linear in the logarithm of the strain rate. While this form is often adequate, most materials exhibit a bi-linear[3] dependence of strength on the logarithm of the strain-rate. Figure 1 shows the strain-rate data for ASTM A36 hot rolled steel obtained from Torsional Split Hopkins Bar[4] experiments, as reported by Battelle (Seidt, 2005), and a standard Johnson-Cook model fit to the data. Each data point represents the effective stress at an effective plastic strain value of 10%. The quality of the data fit is indicated by the RMS value, i.e. square root of the average squared difference between the data and regression value:

$$\mathrm{RMS} = \sqrt{\frac{\sum_{i=1}^{m}\left[Y_i - f(x_i)\right]^2}{m}}$$

Where Y_i is the data at point x_i and $f(x_i)$ is the corresponding value of the regression function.

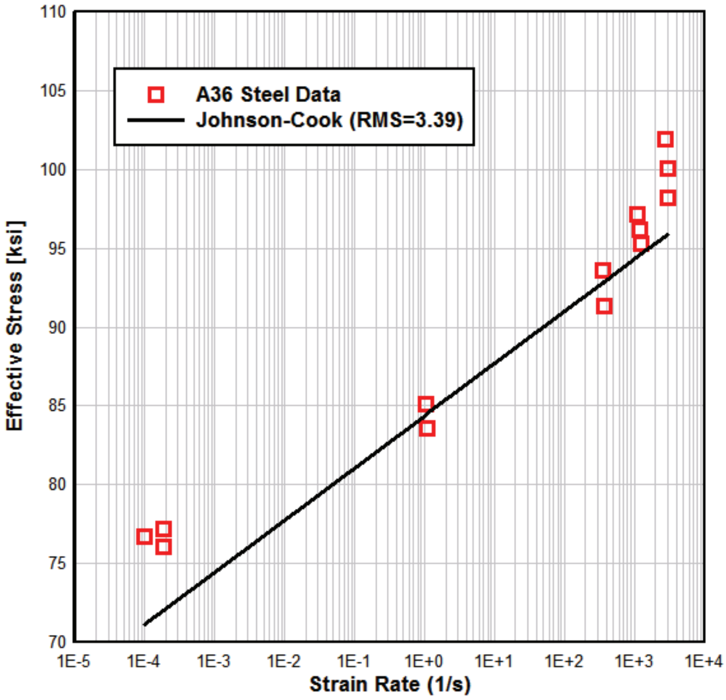

Figure 1 Strain-rate data for A36 steel with standard Johnson-Cook model fit.

The choice of the parameter $\dot{\varepsilon}_0 = 1.0\ \mathrm{sec}^{-1}$, in **Fehler! Verweisquelle konnte nicht gefunden werden.**, is sometimes made as a matter of convenience, or possibly due to a misunderstanding of the role of this parameter. It is often thought this parameter simply plays the role of making the time units in the strain-rate term non-dimensional. The important part of selecting this parameter is to note it must be consistent with the choices of the yield and hardening parameters, i.e. A and B. If the parameters A and B are determined from the quasi-static effective stresses versus effective plastic strain data, then the parameter $\dot{\varepsilon}_0$ should be set to the value of the effective plastic strain-rate used in

[3] The transition from one linear segment to the other typically occurs at strain-rates where the testing techniques, i.e. Split Hopkins Bar (SHB) with uniaxial stress, or flyer plates with uniaxial strain, change to span the strain-rate range.
[4] See for example Gilat (2000).

the quasi-static test, e.g. $\dot{\varepsilon}_0 = 10^{-4}\ \sec^{-1}$. If however $\dot{\varepsilon}_0 = 1.0\ \sec^{-1}$ is selected, then the previously determined values of A and B need to be suitable modified. As an illustration, the Johnson-Cook model is first evaluated using the parameters in **Fehler! Verweisquelle konnte nicht gefunden werden.** at an effective plastic strain of zero, $\varepsilon^p_{eff} = 0$, and the quasi-static effective plastic strain rate of $\dot{\varepsilon}^p_{eff} = 10^{-4}\ \sec^{-1}$, and then again with the same parameters except $\dot{\varepsilon}_0 = 10^{-4}\ \sec^{-1}$:

$$\dot{\varepsilon}_0 = 10^{-4}\ \sec^{-1} \rightarrow \sigma_Y = 41.5\ \text{ksi}$$
$$\dot{\varepsilon}_0 = 1.0\ \sec^{-1} \rightarrow \sigma_Y = 35.0\ \text{ksi}$$

This illustration indicates that using the parameter $\dot{\varepsilon}_0 = 1.0\ \sec^{-1}$ with the parameters A and B determined from quasi-static testing, results in a Johnson-Cook model fit that *under predicts* the static response; this under prediction of the static response can be seen in Figure 1.

3.1 Huh-Kang Rate Form

Huh and Kang (2002) proposed a strain-rate form that is quadratic in the logarithm of the effective plastic strain rate

$$1 + C_1 \ln \dot{\varepsilon} + C_2 \left(\ln \dot{\varepsilon}\right)^2 \quad (1)$$

as a two parameter replacement for the linear form used in the standard Johnson-Cook model.

3.2 Allen-Rule-Jones Rate Form

Allen et al. (1997) proposed a strain-rate form that is an exponential of the effective plastic strain rate

$$\dot{\varepsilon}^c \quad (2)$$

as an alternate one parameter form used in the standard Johnson-Cook model.

3.3 Cowper-Symonds Rate Form

NAVEODTECHDIV also reported the use of the popular Cowper and Symonds (1958) rate form

$$1 + \left(\frac{\dot{\varepsilon}^p_{eff}}{C}\right)^{\frac{1}{P}} \quad (3)$$

as a two parameter exponential replacement for the standard Johnson-Cook model.

3.4 RATE FORM REGRESSION PARAMETERS

The strain-rate data for A36 steel, shown previously in Figure 1, was fit to the various rate forms using two values of the parameter $\dot{\varepsilon}_0 = 1.0$ and $1.54 \times 10^{-4}\ \sec^{-1}$. The former value corresponds to the value in **Fehler! Verweisquelle konnte nicht gefunden werden.**, and the latter value is the average effective plastic strain rate for the three quasi-static tests, i.e. data in the lower left corner of Figure 1. The regression coefficients for the various rate forms are provided in Table 1.

Table 1 Regression parameters for various rate forms using two values for $\dot{\varepsilon}_0$.

	$\dot{\varepsilon}_0 = 1.0\ \text{sec}^{-1}$	$\dot{\varepsilon}_0 = 1.54 \times 10^{-4}\ \text{sec}^{-1}$
Johnson-Cook	$C = 1.705 \times 10^{-2}$	$C = 1.622 \times 10^{-2}$
Huh-Kang	$C_1 = 1.613 \times 10^{-2}$ $C_2 = 6.646 \times 10^{-4}$	$C_1 = 2.149 \times 10^{-3}$ $C_2 = 9.112 \times 10^{-4}$
Allen-Rule-Jones	$C = 1.731 \times 10^{-2}$	$C = 1.451 \times 10^{-2}$
Cowper-Symonds	$C = 3.335 \times 10^{5}\ \text{sec}^{-1}$ $P = 2.849$	$C = 3.335 \times 10^{5}\ \text{sec}^{-1}$ $P = 4.203$

Figure 2 shows the A36 steel strain-rate data, shown previously in Figure 1, along with the regression fits for the four rate forms. For both values of the parameter $\dot{\varepsilon}_0$, the Huh-Kang form provides the best fit to the data, i.e. lowest values of RMS. The RMS value is lower for all four strain-rate forms when the parameter $\dot{\varepsilon}_0 = 1.54 \times 10^{-4}\ \text{sec}^{-1}$ is used. The improved fit for the Cowper-Symonds strain-rate form, shown on the left in Figure 2, is not due to the parameter $\dot{\varepsilon}_0$, since this rate form does not involve this parameter, but rather the improved fit appears to be the result of first normalizing the ordinate values by the average quasi-static value of the effective stress, i.e. 76.6 ksi.

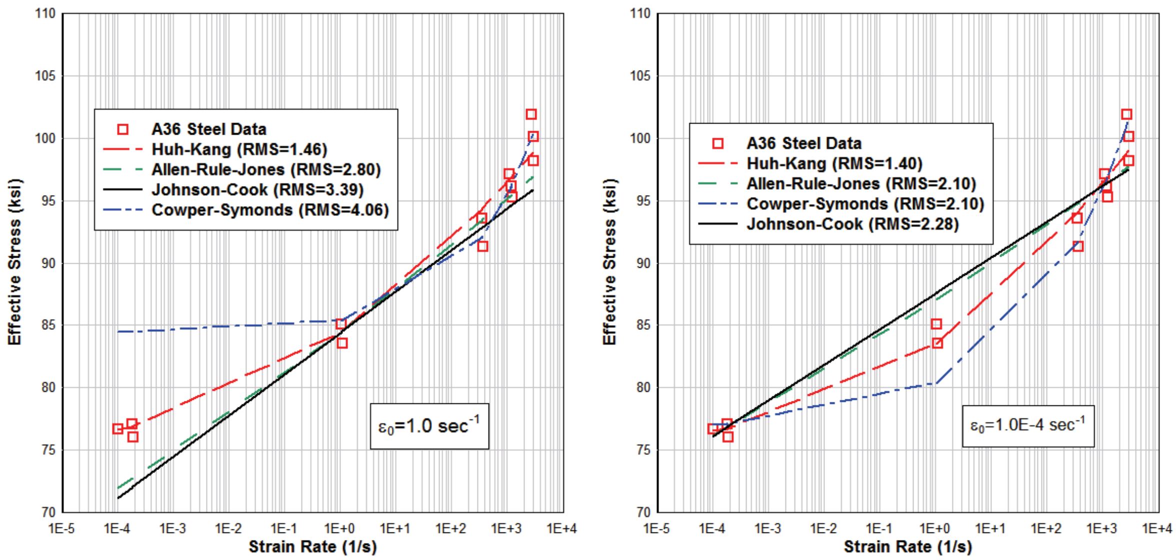

Figure 2 Comparison of four strain-rate forms fit to A36 steel data using two values of the parameter $\dot{\varepsilon}_0$.

3.5 Summary

The Huh-Kang rate form provides a significant improvement over the standard Johnson-Cook rate form, and the other two rate forms considered, for the A36 steel data presented. Improvements in all four rate forms can be obtained by suitable selection of the parameter $\dot{\varepsilon}_0$, and in the case of the Cowper-Symonds form, normalizing the effective stress to the quasi-static value.

It is important to note that the data shown in Figure 2 represent a single point from each of the effective stress versus effective plastic strain strain-rate curves. In a subsequent section, comparisons between the full span of the stress-strain data and simulation results are presented. The conclusion there is that the standard Johnson-Cook rate form provides the best overall agreement with the A36 steel data.

4 LS-DYNA Viscoplastic Plastic Rate Form Implementation

The three optional strain-rate forms described above, i.e. Huh-Kang, Allen-Rule-Jones, and Cowper-Symonds, were implemented in LS-DYNA Version 971 as options to Material Model 15, i.e. *MAT_JOHNSON_COOK. The optional rate forms are *only available* under the material model's viscoplastic rate option, i.e. input parameter VP=1.

5 Comparisons of Strain-Rate Forms with A36 Steel Data

The Battelle A36 steel strain-rate data consists of repeat tests at five strain-rates in the range of 2×10^{-4} to 3000 $\sec^{-1}$, see Figure 1. For the present purpose of comparison, only three of these strain-rates are considered: the *quasi-static* strain-rate of $2\times10^{-4}\ \sec^{-1}$, the *nominal* strain-rate of $1.0\ \sec^{-1}$ used to determined the yield and hardening parameters A and B, and the *moderately high* strain-rate of 360 $\sec^{-1}$. The effective stress versus effective plastic strain A36 data for these three strain rates are shown in Figure 3.

The yield and hardening parameters A and B reported previously in **Fehler! Verweisquelle konnte nicht gefunden werden.**, were obtained via regression of the effective stress versus effective plastic strain A36 steel data at a nominal strain rate of 1.0 $\sec^{-1}$, i.e. $\dot{\varepsilon}_0 = 1.0\ \sec^{-1}$. Because this strain rate is the typical (nominal) input value for this parameter, it is a convenient strain-rate to use in making comparisons of the A36 data with the results from the LS-DYNA implementation of the optional strain-rate forms.

An alternate value of the strain-rate parameter, $C = 1.622\times10^{-2}$, was obtained by normalizing the A36 steel strain-rate data using the average quasi-static strain-rate, i.e. $\dot{\varepsilon}_0 = 1.54\times10^{-4}\ \sec^{-1}$. To properly use this alternative strain-rate parameter value in the Johnson-Cook model requires modification of the yield and hardening parameters A and B by calibrating them to the quasi-static stress-strain data. A comparison of the nominal (**Fehler! Verweisquelle konnte nicht gefunden werden.**) and quasi-static parameters obtained by calibrating the yield and hardening parameters to the quasi-static stress-strain data are summarized in Table 2.

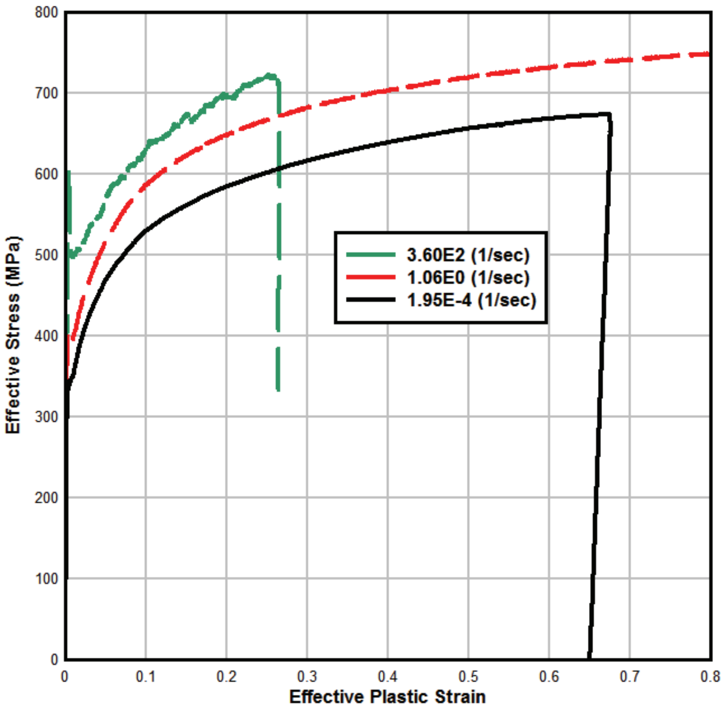

Figure 3 A36 steel stress-strain data at three strain rates.

Table 2 Comparison of Johnson-Cook parameters provided by NAVEODTECHDIV with those obtained by calibration to the quasi-static data.

	A (ksi)	B (ksi)	N	C	$\dot{\varepsilon}_0\ (\mathrm{sec}^{-1})$
Nominal	41.50	72.54	0.228	0.0171	1.0
Quasi-Static	36.25	76.19	0.328	0.0162	1.9×10^{-4}

6 LS-DYNA Johnson-Cook Model Simulations

In the following three subsections effective stress versus effective plastic strain comparisons are presented for the A36 steel with corresponding results from the modified LS-DYNA Johnson-Cook model that includes the three optional strain-rate forms, e.g. Huh-Kang, Allen-Rule-Jones, and Cowper-Symonds. A single solid hexahedral element (unit cube) is used to simulate a uniaxial stress loading state with prescribed nodal velocities providing the desired strain-rate. In addition to the yield, hardening, and strain-rate parameters summarized in **Fehler! Verweisquelle konnte nicht gefunden werden.** though Table 2, the following table summarizes the remaining Johnson-Cook model parameters. The elastic bulk modulus provided in Table 3 was used as the only parameter, C1, in the linear equation-of-state, i.e. *EOS_Linear_Polynomial. Additionally, none of the Johnson-Cook damage parameters were active, and a larger specific heat[5] was used to minimize temperature effects, as the present comparison focuses on comparisons of the strain-rate models, and validation of theses models in the modified LS-DYNA Johnson-Cook model.

Table 3 Summary of additional Johnson-Cook model parameters.

Parameter	Value	Description
ρ	$7.85\times10^{-3}\ \mathrm{g/mm^3}$	Density
G	$76.9\ \mathrm{GPa}$	Shear modulus

[5] A specific heat of 4568 J/kg-K was used in the calculations.

K	166.6 GPa	Bulk modulus
M	0.917	Temperature parameter
T_M	1773 °K	Melt temperature
T_R	293 °K	Reference temperature
C_p	486 J/kg-°K	Specific heat
PC	-1.0×10^6 MPa	Pressure cutoff
SPALL	1.0	Spall type
IT	1.0	Iteration option

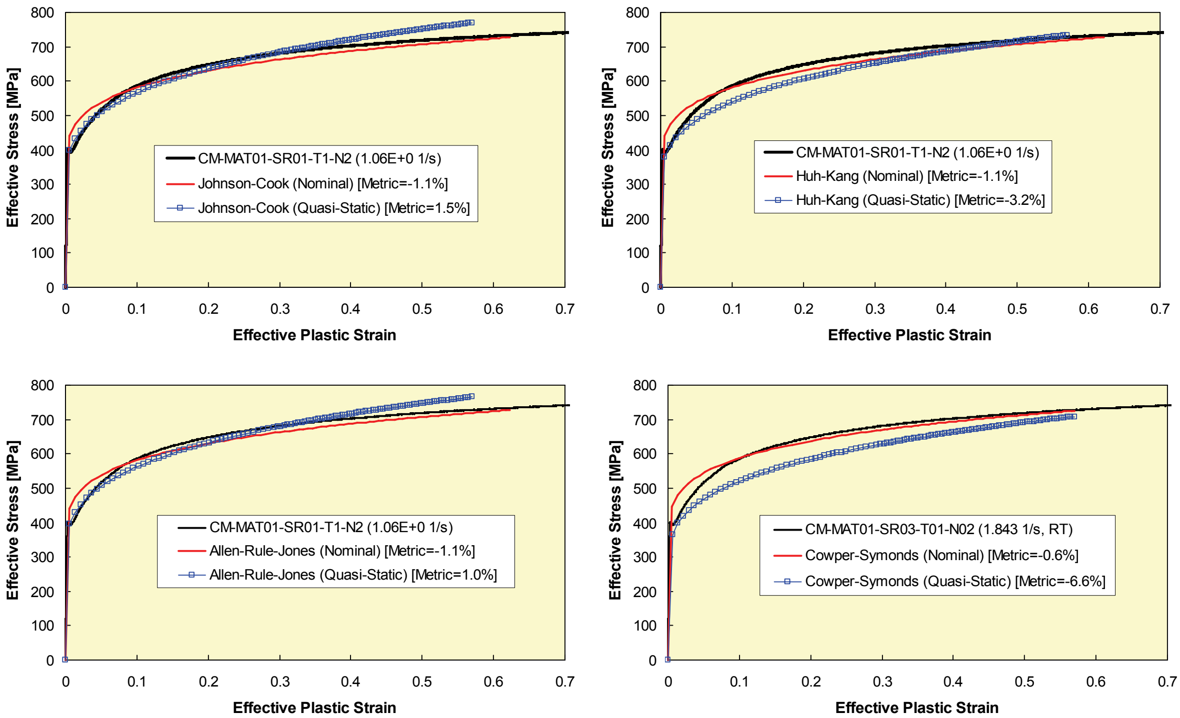

Figure 4 Comparison of four strain-rate forms with A36 steel stress-strain data at a nominal strain-rate of 1.0 $\sec^{-1}$

6.1 Nominal Strain Rate Comparisons

Figure 4 compares the four strain-rate forms for the two sets of calibration parameters with the A36 steel effective stress versus effective plastic strain data at a nominal strain rate of 1 $\sec^{-1}$. To provide a quantitative assessment of the comparisons, a magnitude metric is used to compare the data with the corresponding computed result; see Sprague & Geers (2004). A metric value of zero indicates perfect agreement, with larger values indicating less agreement. The metric comparisons are summarized in Figure 5. The agreement for both parameter sets are quite good, the notable exception being the Cowper-Symonds form for the quasi-static parameter set, i.e. using $\dot{\varepsilon}_0 = 1.54\times10^{-4}\ \sec^{-1}$. For this strain-rate of 1.0 $\sec^{-1}$, the nominal parameter set (**Fehler! Verweisquelle konnte nicht gefunden werden.**) provides the better overall comparison, although almost no distinction can be made as to which strain-rate form should be preferred.

It may at first be surprising that three of the four rate forms provide identical metric values for the nominal parameter set, i.e. magnitude metric = -1.1% for Johnson-Cook, Huh-Kang & Allen-Rule Jones strain-rate forms. However, since the prescribed effective plastic strain-rate is equal to the normalizing strain rate, i.e. $\dot{\varepsilon}^{p}_{eff} = \dot{\varepsilon}_0 = 1.0$, inspection of the standard Johnson-Cook rate form and Equations (1) and (2) indicates the strain-rate is identically unity for this case, and thus the computed responses are independent of strain-rate.

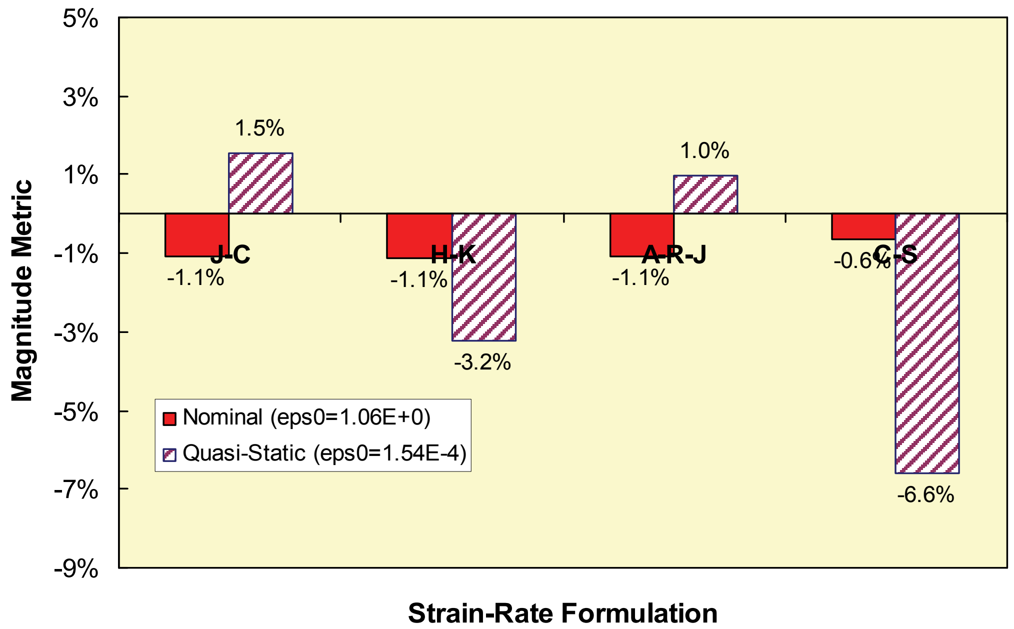

Figure 5 Metric based comparison of four strain-rate forms for A36 steel stress-strain data at a nominal strain-rate of 1.0 $\sec^{-1}$

6.2 Quasi-Static Strain Rate Comparisons

Figure 6 compares the four strain-rate forms for the two sets of calibration parameters with the A36 steel effective stress versus effective plastic strain data at a quasi-static strain rate of $1.95\times10^{-4}\ \sec^{-1}$. As can be seen in these comparisons, and as quantified in the metric comparisons shown in Figure 7, the nominal parameter set (**Fehler! Verweisquelle konnte nicht gefunden werden.**) uniformly over predicts the effective stress for all strain-rate forms. The quasi-static parameter set, i.e. using $\dot{\varepsilon}_0 = 1.54\times10^{-4}\ \sec^{-1}$, uniformly under predicts the effective stress for all strain-rate forms. However, the quasi-static parameter set provides a better agreement with the data than the nominal parameter set by a factor of three, i.e. 3.2 = 8.1% / 2.5%. For both sets of parameters, no distinction can be made as to which strain-rate forms provides the best fit to the A36 steel data.

The *over prediction* of the effective stress by the nominal parameter set is in direct contradiction with calibration of the strain rate parameters shown previously in Figure 2 (left). The nominal parameter set, using $\dot{\varepsilon}_0 = 1.0\ \sec^{-1}$ and corresponding yield and hardening parameters, was expected to provide an *under prediction* for the standard Johnson-Cook and Allen-Rule-Jones strain-rate forms. Only the Cowper-Symonds strain-rate form was expected to produce an over prediction. So the question arises, why does the algorithmic implementation of these four different models provide a nearly uniform over prediction of the response when compared to the quasi-static data?

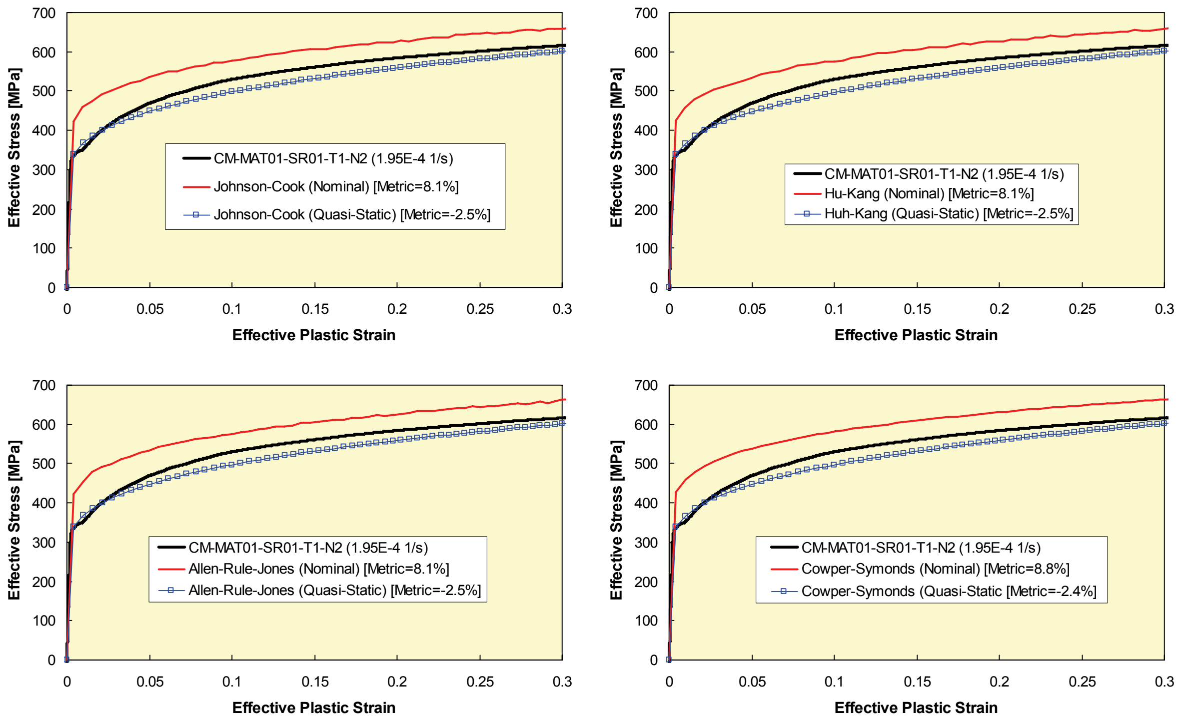

Figure 6 Comparison of four strain-rate forms with A36 steel stress-strain data at a quasi-static strain-rate of $1.95\times10^{-4}\ \text{sec}^{-1}$.

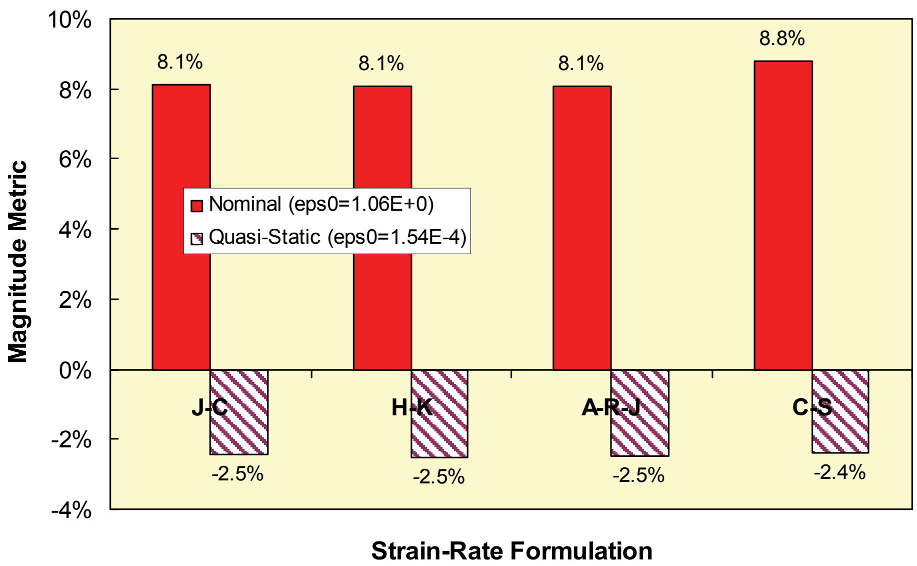

Figure 7 Metric based comparison of four strain-rate forms for A36 steel stress-strain data at a quasi-static strain-rate of $1.95\times10^{-4}\ \text{sec}^{-1}$.

The basic reason the four strain-rate forms, calibrated with the nominal parameters, over predicting the quasi-static response is that the plasticity algorithm, used to update the stress given the strain increment, cannot provide a plasticity solution for a stress state inside the nominal yield surface. In the plasticity algorithm, the increment in strain is used to compute the elastic trail stress, this in turn is compared to the *rate-independent* yield function to determine if plasticity has occurred. When yielding occurs, the algorithm seeks to determine the amount of the strain increment that was plastic via an iteration process that changes the estimate of the plastic strain increment until the stress state is on the yield surface. An underlying assumption of the algorithm is that the new stress state has to be *greater than*, or equal, to the previous state since loading has occurred.

However, for the nominal set of parameters the new stress state is *less than* the previous stress state. So the plasticity algorithm adjusts the plastic strain increment, and corresponding plastic strain-rate, until the stress return to the *rate-independent* yield function. Thus the four strain-rate forms, calibrated using the nominal parameters, predict an effective stress versus effective plastic strain response that is essentially equal to the nominal $\dot{\varepsilon}_0 = 1.0\ \sec^{-1}$ model fit. Recall from the above nominal strain-rate case, that this model fit is independent of the strain rate for the three strain-rate forms that use $\dot{\varepsilon}_0$, and thus the metrics are identical in this quasi-static case, as they were in the previous case for the nominal strain rate of $1.0\ \sec^{-1}$.

The above plasticity algorithm argument is confirmed by noting the simulation results from the quasi-static calibration set replicate the quasi-static response quite well, i.e. metric value of -2.5%, as is to be expected.

6.3 Moderately High Strain Rate Comparisons

While the preceding nominal and quasi-static strain-rate studies were informative in illustrating the role of $\dot{\varepsilon}_0$, and the necessity of calibrating the yield and hardening parameters to the quasi-static data, the present moderately high strain-rate case provides some insight into which rate form best replicates the A36 data at elevated strain-rates.

Figure 8 compares the four strain-rate forms for the two sets of calibration parameters with the A36 steel effective stress versus effective plastic strain data at a moderately high strain rate of $360\ \sec^{-1}$. As in the case of the quasi-static comparisons, the nominal parameter set uniformly over predicts the effective stress for all strain-rate forms; see the metric comparisons shown in Figure 9. The quasi-static parameter set, i.e. using $\dot{\varepsilon}_0 = 1.54 \times 10^{-4}\ \sec^{-1}$, generally also over predicts the effective stress, with the exception of an under prediction in the case of the Huh-Kang rate form. For all strain-rate forms, the quasi-static parameter set provides a better agreement with the data than the nominal parameter set. For both sets of parameters, the standard Johnson-Cook strain-rate form provides the best fit to the A36 steel data at this moderately high strain rate.

6.4 Summary

The above effective stress versus effective plastic strain comparisons with A36 steel data suggests the preferred form for the strain-rate term in the Johnson-Cook model is the standard Johnson-Cook rate form, followed closely by the Allen-Rule-Jones form. This assessment is based on the recommendation that the yield and hardening parameters are calibrating to the quasi-static effective stress versus effective plastic strain data rather than using a calibration at a nominal strain rate of 1.0 $\sec^{-1}$.

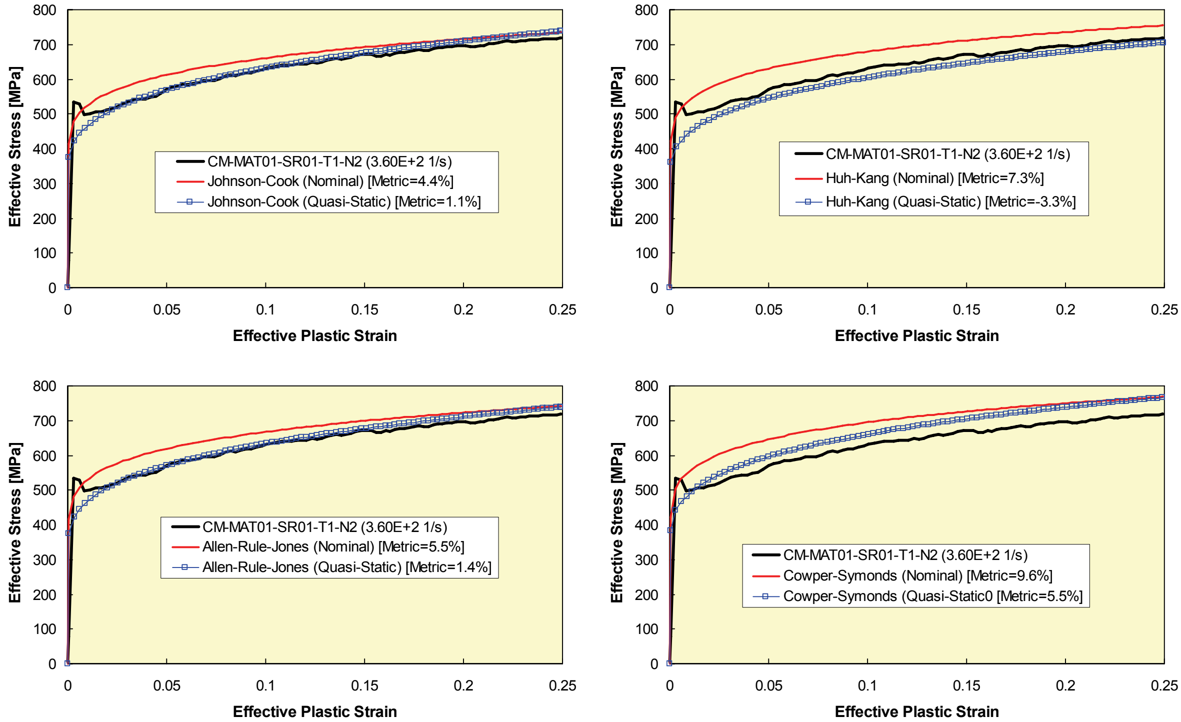

Figure 8 Comparison of four strain-rate forms with A36 steel stress-strain data at a moderately high strain-rate of $360\ \sec^{-1}$.

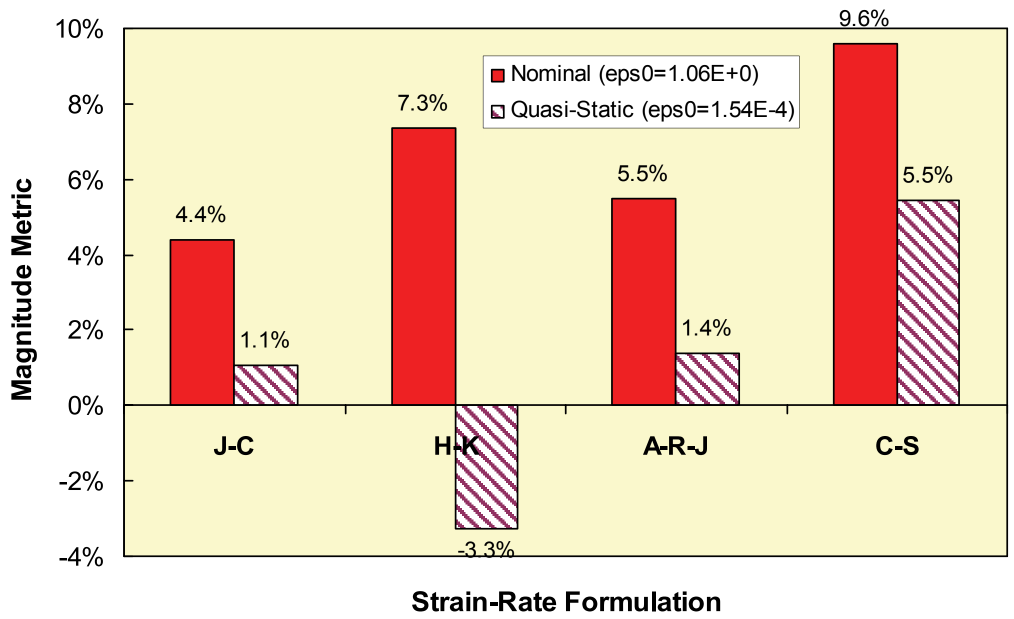

Figure 9 Metric based comparison of four strain-rate forms for A36 steel stress-strain data at a quasi-static strain-rate of $360\ \sec^{-1}$.

7 Conclusions

Effective stress versus effective plastic strain strain-rate data for A36 steel was compared with single element results from LS-DYNA using four strain-rate forms implemented in a modified the Johnson-

Cook constitutive model. The comparisons indicate the standard Johnson-Cook strain-rate form provides the best overall comparison with the data, with the Allen-Rule-Jones form a close second.

The main points conveyed in this document are:

1. The constitutive model parameters should be calibrated to the quasi-static data, rather than using a nominal strain of 1.0 $\sec^{-1}$. If the model parameters are calibrated to other than the quasi-static data, the plasticity algorithm will provide an effective stress equivalent to the calibrated strength curve for all strain-rates less than the calibration strain-rate. The result will be that parts of the model where the stain-rates are low, but the strains are sufficient to yield the material, will exhibit a strength greater than the quasi-static strength.
2. The role, and importance, of the strain-rate normalization parameter $\dot{\varepsilon}_0$ is explained and demonstrated. This is *not* simply a parameter for making the effective plastic strain-rate non-dimensional, as is often incorrectly cited, but this parameter must be specified as the effective plastic strain rate of the quasi-static testing used to calibrate the yield and hardening parameters.

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