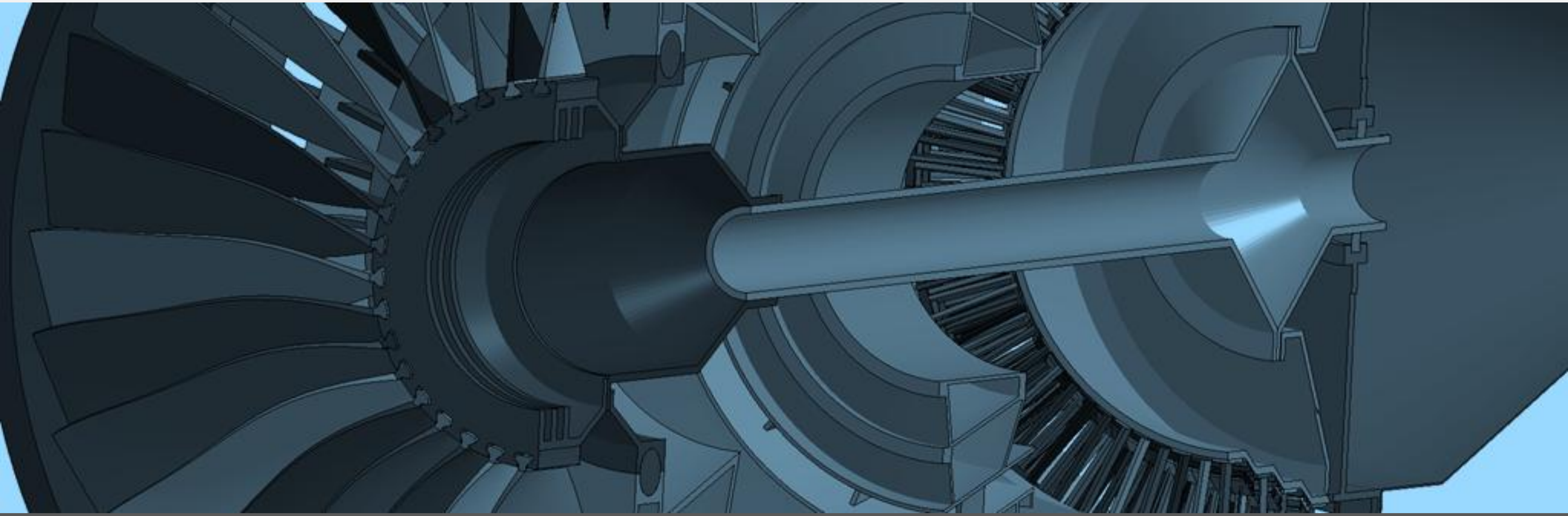

Optimization of Newmark-Euler Time-Integration Parameters for a Stable and Efficient Implicit Simulation of Rotating Elastic Structures

LS-DYNA Forum 2018, Bamberg

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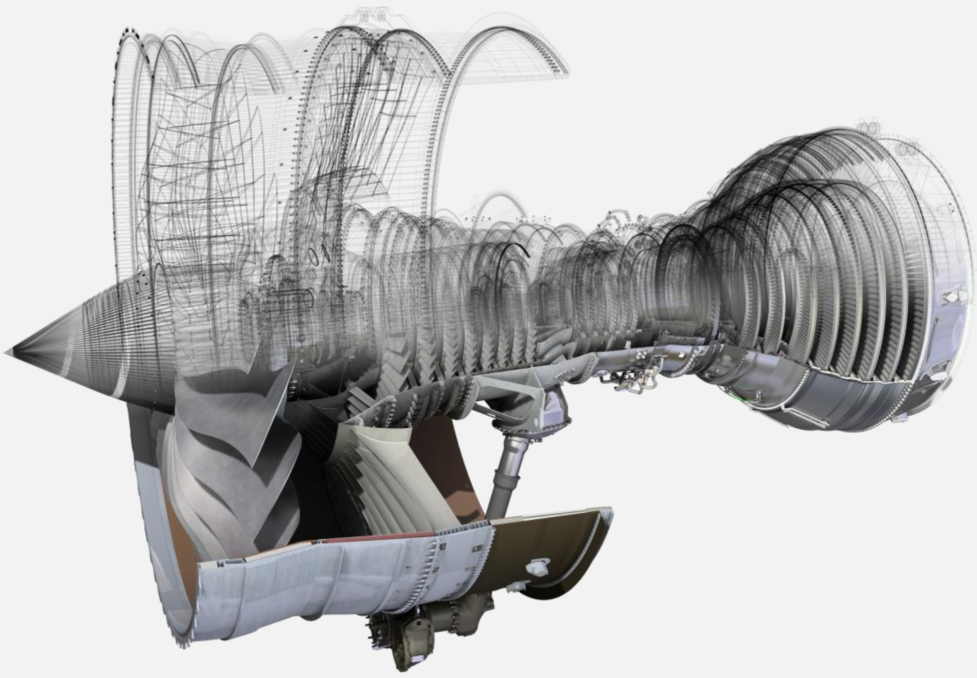
Outline

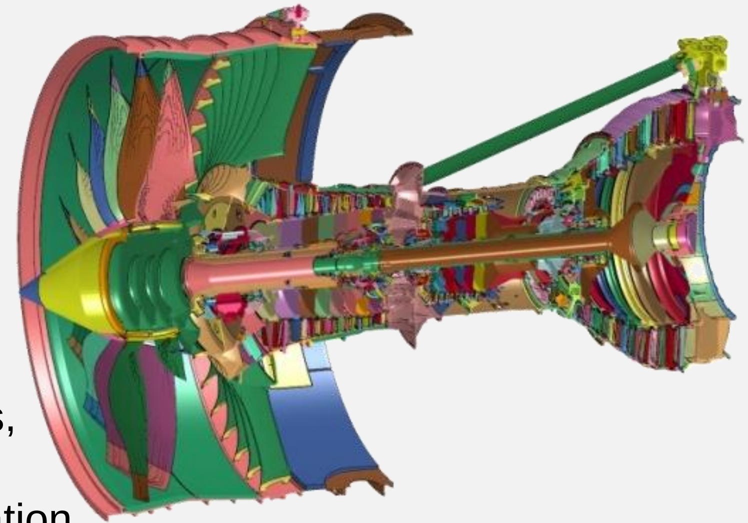


- Motivation
- Instability problems of implicit time-integration
- Reasons for appearance of instabilities
- Complex FEM example
- Stability considerations of time-integration algorithms
- Optimization of Newmark-Euler time-integration parameters
- Computational results with optimized parameters
- Summary

Motivation

Objective

- In aerospace industry more and more detailed FEM models are used for prediction of engine behavior
- Goal is the thermo-mechanical simulation of a running engine with almost no idealizations or simplifications over a time-span of a few seconds



⇩

Dynamic models with millions of DOF

- Many issues have to be solved (contact problems, modeling of bearings, hourglassing, efficient parallelization, model decomposition, time integration, …)

⇨ **What kind of time integration scheme should be used?**

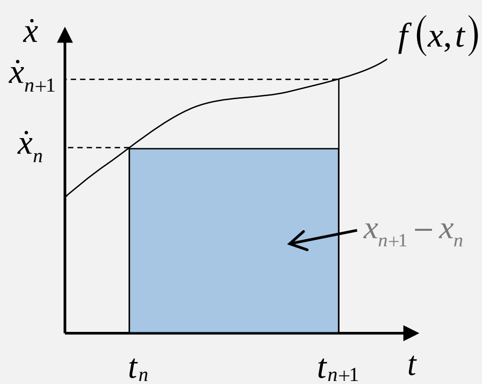
Implicit vs. explicit time integration

- Equation of motion has to be integrated two times for solving the boundary value problem: $M\ddot{x}(t) + D\dot{x}(t) + Kx(t) = F(t)$
- For simplicity we assume to solve the equation $\dot{x} = f(x,t)$ by:

Explicit time integration

E.g. Euler-Forward method:

$$x_{n+1} = x_n + f(x_n, t_n) \cdot \Delta t$$



Solution has to be determined by solving linear or nonlinear equations (e.g. by Newton iterations)

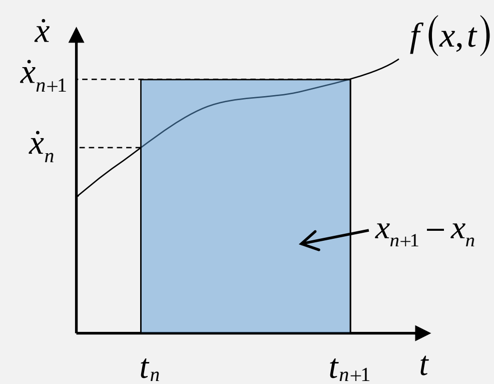
○ Known quantity

○ Unknown quantity

Implicit time integration

E.g. Euler-Backward method:

$$\rightarrow x_{n+1} = x_n + f(x_{n+1}, t_{n+1}) \cdot \Delta t$$



Because of Newton iteration and therefore computation of "exact" function value at t_{n+1} with implicit time integration much bigger time steps are possible than in an explicit time integration!

Motivation

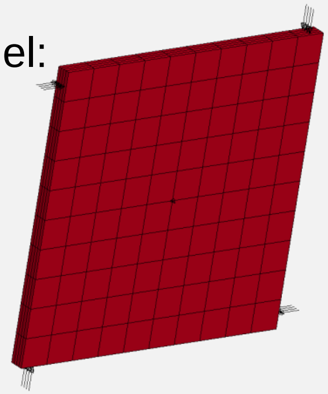
- **Implicit** time integration algorithms can be constructed such that they are **unconditionally stable** (solution will not grow uncontrolled for arbitrary big time steps)

- **Explicit** time integration algorithms are only **conditionally stable** (solution will grow uncontrolled for too big time steps)
- For aero-engine models the critical time step for an explicit analysis is in the order of **10^{-8}s**
- Due to these extremely small time steps, e.g. analysis of contact behavior is much simpler
- But since even the explicit analysis of **40ms** of engine model takes **a few weeks on thousands of cores**, the **explicit analysis is not an option** for the simulation of a running engine over a few seconds

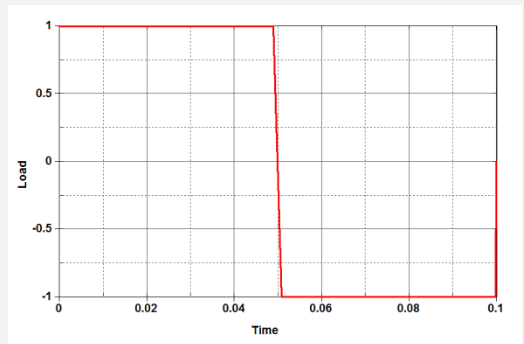
Instability Problem of Implicit Time-Integration

Implicit Simulation of Spinning Plate with Newmark algorithm

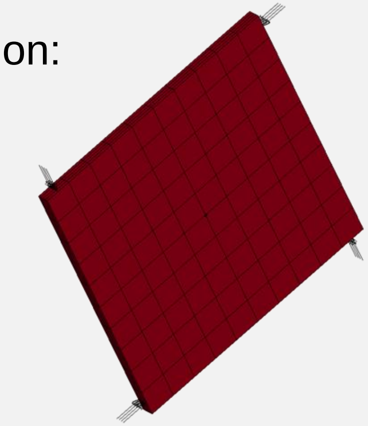
Model:

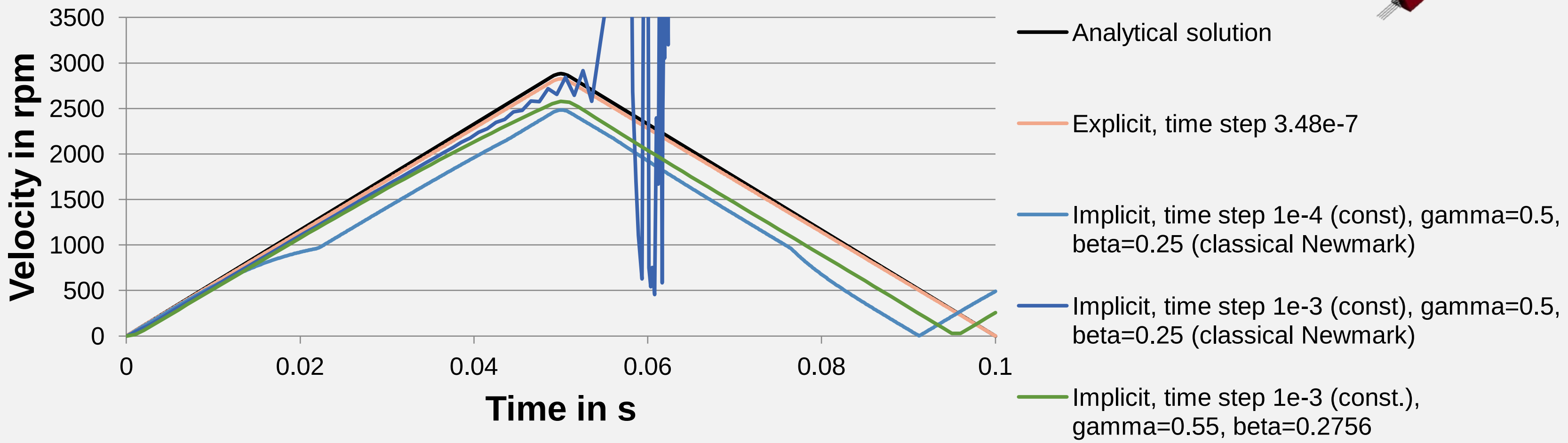


Load curve:



Simulation:

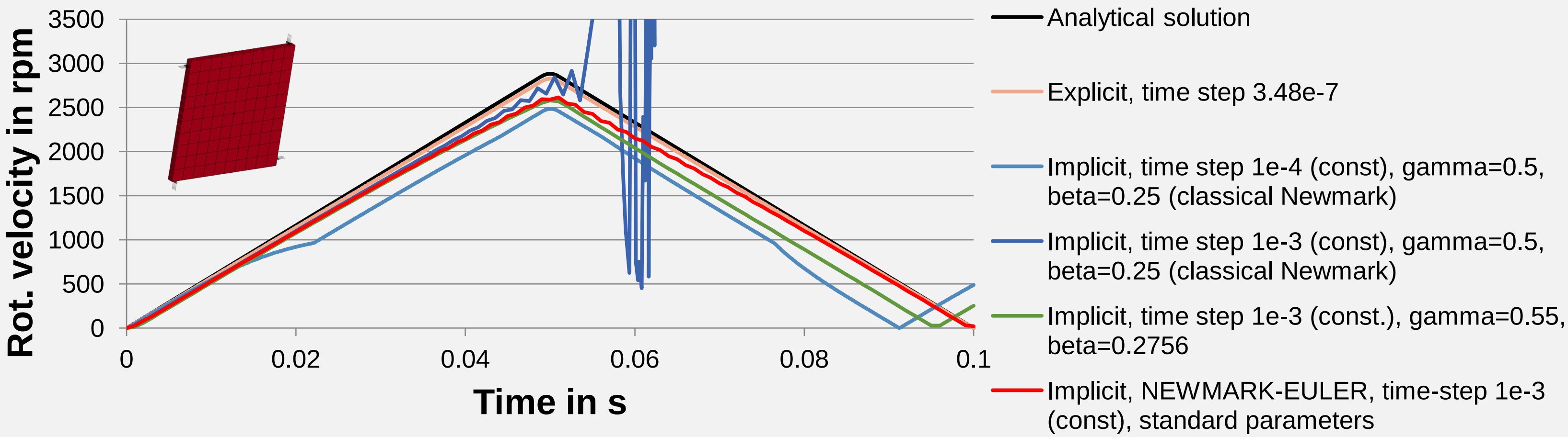




- Newmark algorithm becomes instable with standard parameters
- Change of parameters improves situation but new parameters are only valid for particular problem

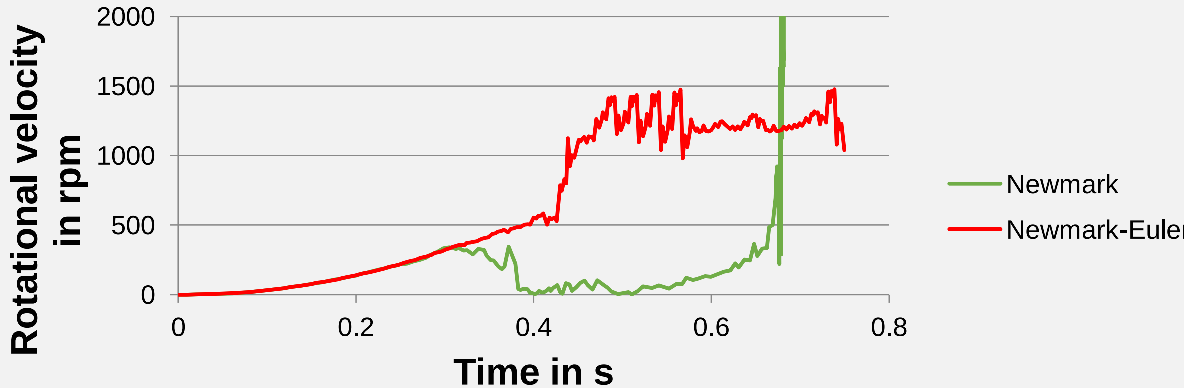
Instability Problem of Implicit Time-Integration

- Literature suggests more advanced time-integration methods like Newmark-Euler:



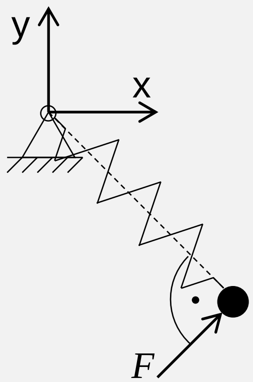
- Newmark-Euler works better than classical Newmark algorithm but also becomes instable in certain situations (automatic time-step control is used):

Driven by pressure load on blades

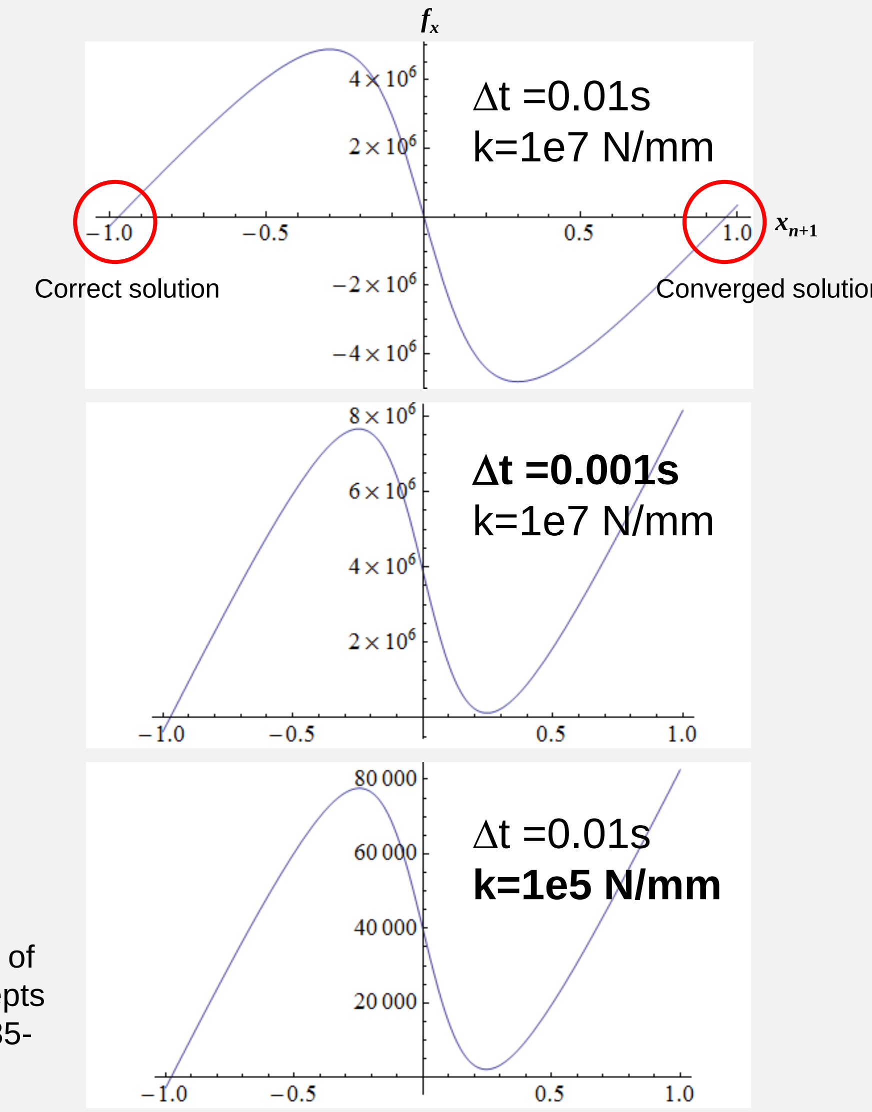


Reasons for Appearance of Instabilities

- Under certain conditions (depending on model stiffness and time-step size) more than one equilibrium solution can appear
- Newton or Quasi-Newton algorithm converges into this "wrong" equilibrium state



- Number of equilibrium solutions is influenced by time-step size and model stiffness
- Instabilities can be detected/avoided by improved time-step control
- For details please refer to [1]

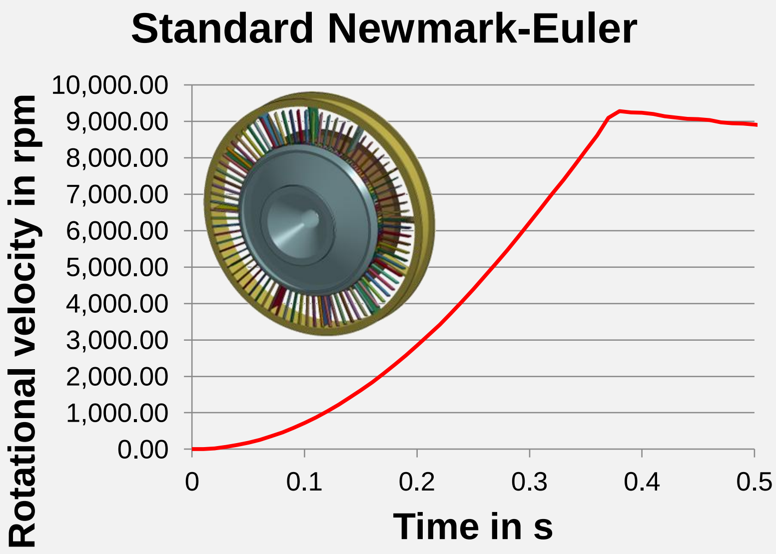


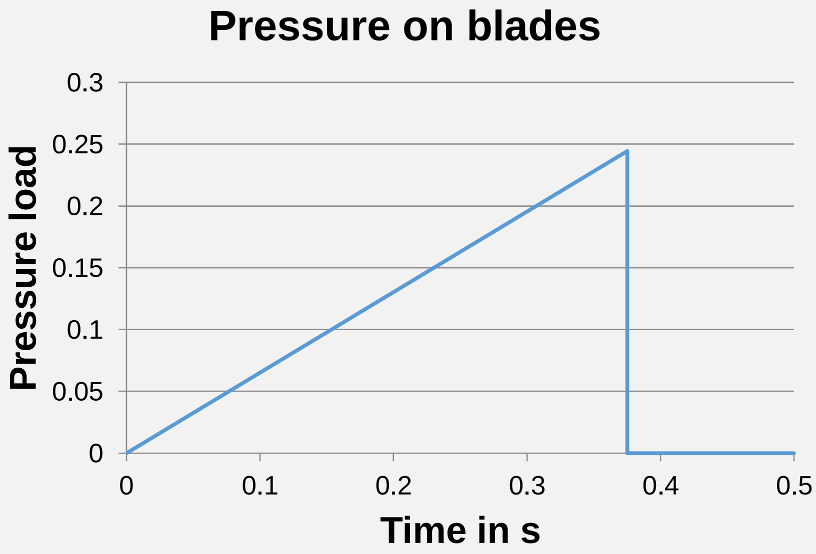
[1] Kober, M. und Kühhorn, A. (2018): Stable implicit time-integration of flexible rotating structures - explanation for instabilities and concepts for avoidance. Applied Mathematical Modelling, Volume 60, pp 235-243.

Complex FEM Example

- In general Newmark-Euler (or Composite) time-integration suffers the same instability problems like Newmark time-integration
- But numerical situation for Newmark-Euler scheme is slightly better and it is a bit faster for rotating structures (additional advantage of damping of higher frequencies)

- Complex FEM example consists of a simplified turbine model including contact in the bearing region, which is loaded by pressure on the blades





→ Due to numerical damping of time-integration algorithm the rotational velocity is decreasing

Stability Considerations of Time-integration Algorithms

- Stability analysis is useful to discover possible weak points of an integration algorithm and to show the borders of its applicability in terms of stability
- Many time integration methods (e.g. Newmark, Newmark-Euler) can be written in the form

$$\begin{bmatrix} \ddot{x}_{n+1} \\ \dot{x}_{n+1} \\ x_{n+1} \end{bmatrix} = \mathbf{A} \cdot \begin{bmatrix} \ddot{x}_{n} \\ \dot{x}_{n} \\ x_{n} \end{bmatrix} + \mathbf{L}\, r_{n+1}$$

$\mathbf{A}$ = Matrix of integration approximation
$\mathbf{L}$ = Load operator
r = External load

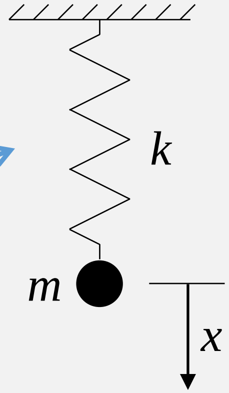
- Matrix A depends on the time integration method (and the mechanical problem), that is used
- An integration method is stable if the **spectral radius** of the matrix $\mathbf{A}$ (depending on Δt/T) is always smaller or equal than 1

Spectral radius of $\mathbf{A}$: $\rho(\mathbf{A}) = \max|\lambda_i|, \quad i = 1, 2, 3$

Stability criterion: $\rho(\mathbf{A}) \leq 1$

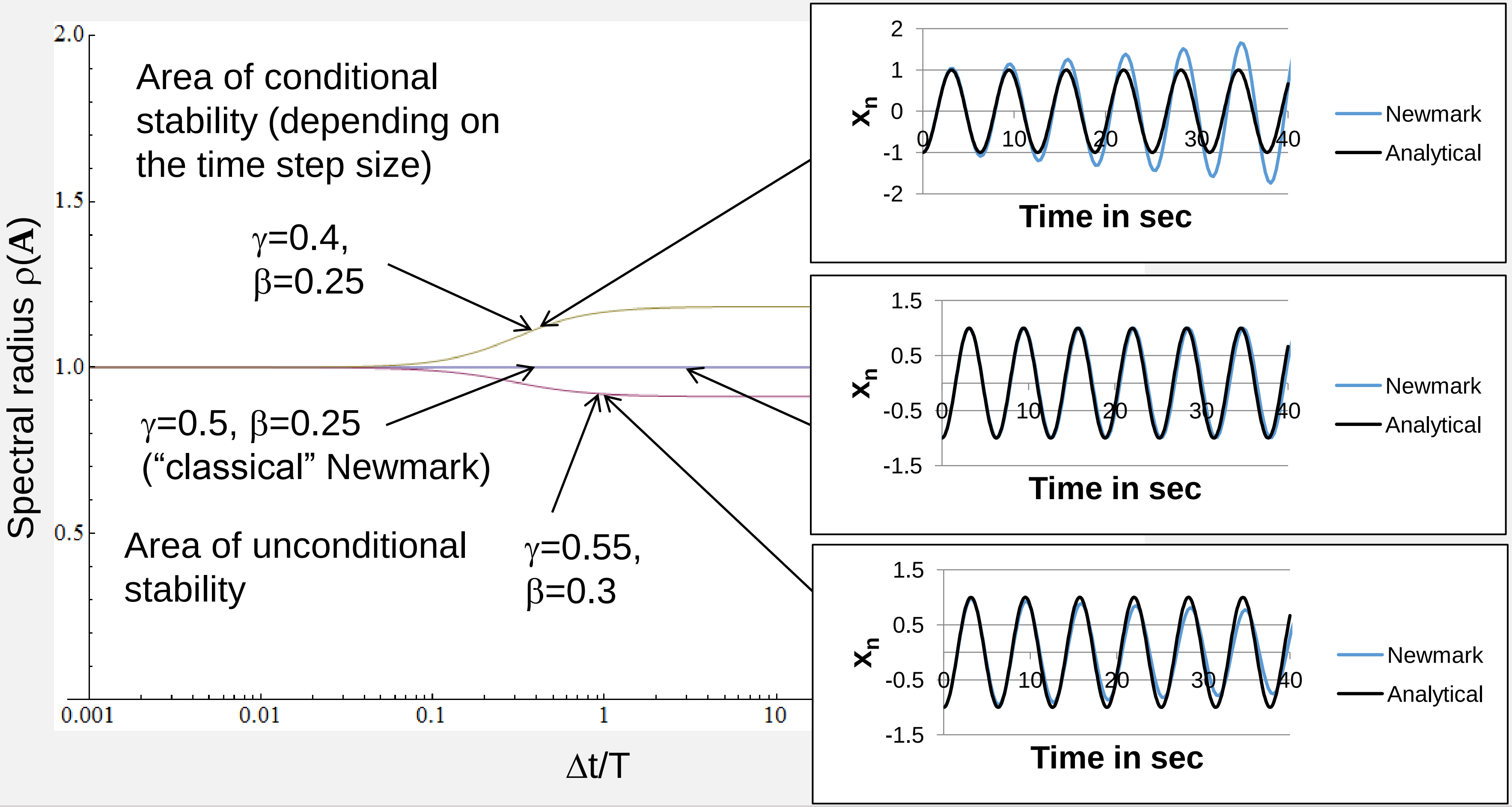
- For stability analysis it is sufficient to consider a simple undamped free vibration problem:

$\ddot{x} + \omega^2 x = 0$ => r = 0



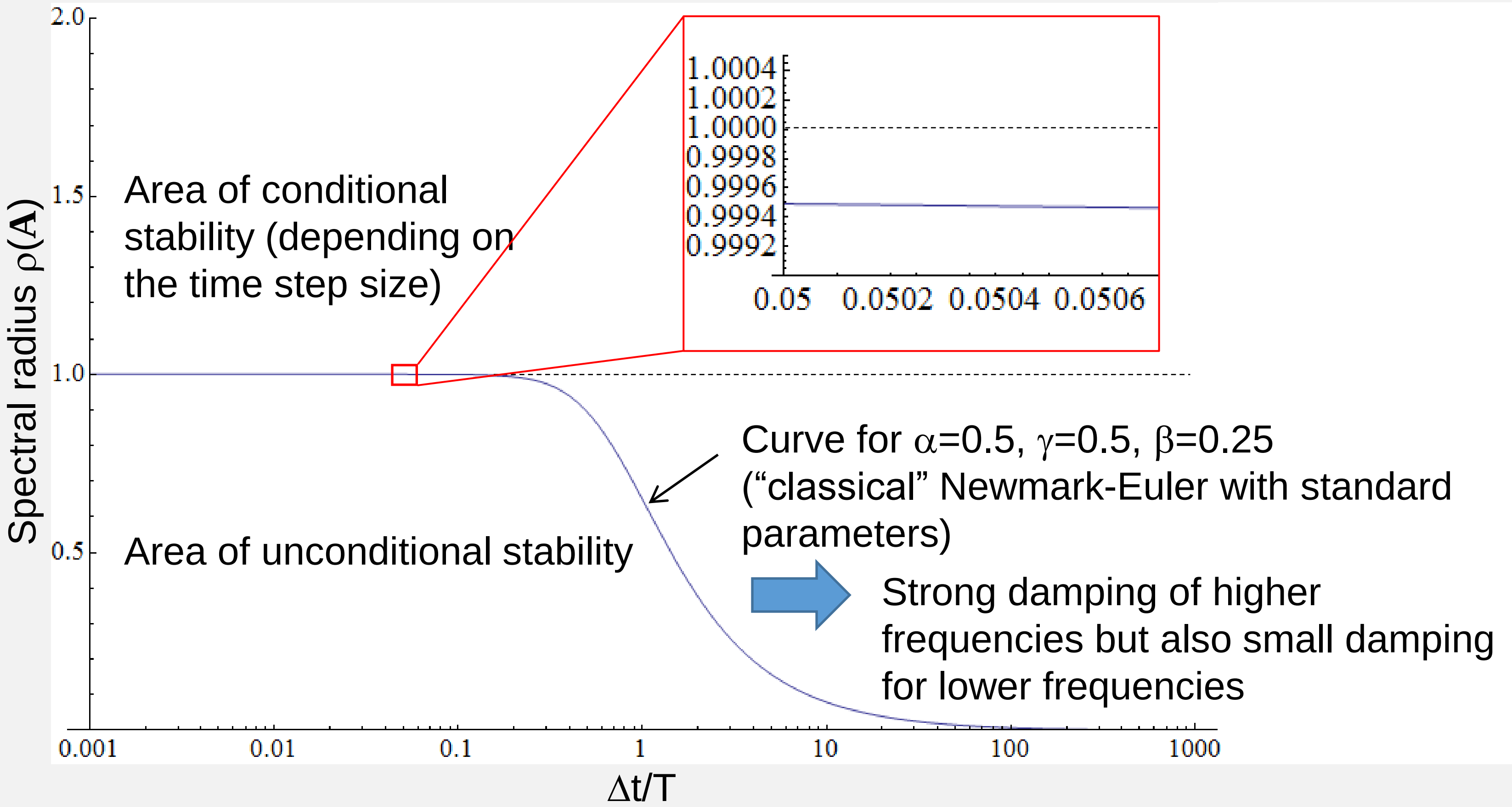
Stability Map of Newmark algorithm

- With T = 2π/ω = 1 it is possible to compute a stability map (plot of spectral radius over Δt/T):



Stability Map of Newmark-Euler algorithm

- With $T = 2\pi/\omega = 1$ it is possible to compute a stability map (plot of spectral radius over $\Delta t/T$):



Optimization of Newmark-Euler Parameters

Design parameters

- Newmark-Euler parameters
  α, γ and β

Objective

- $\max\limits_{\alpha,\gamma,\beta} \int\limits_{0}^{0.1} \rho(\mathbf{A})\, d(\Delta t/T)$

Constraints

- $\max \rho(\mathbf{A}) \leq 1$

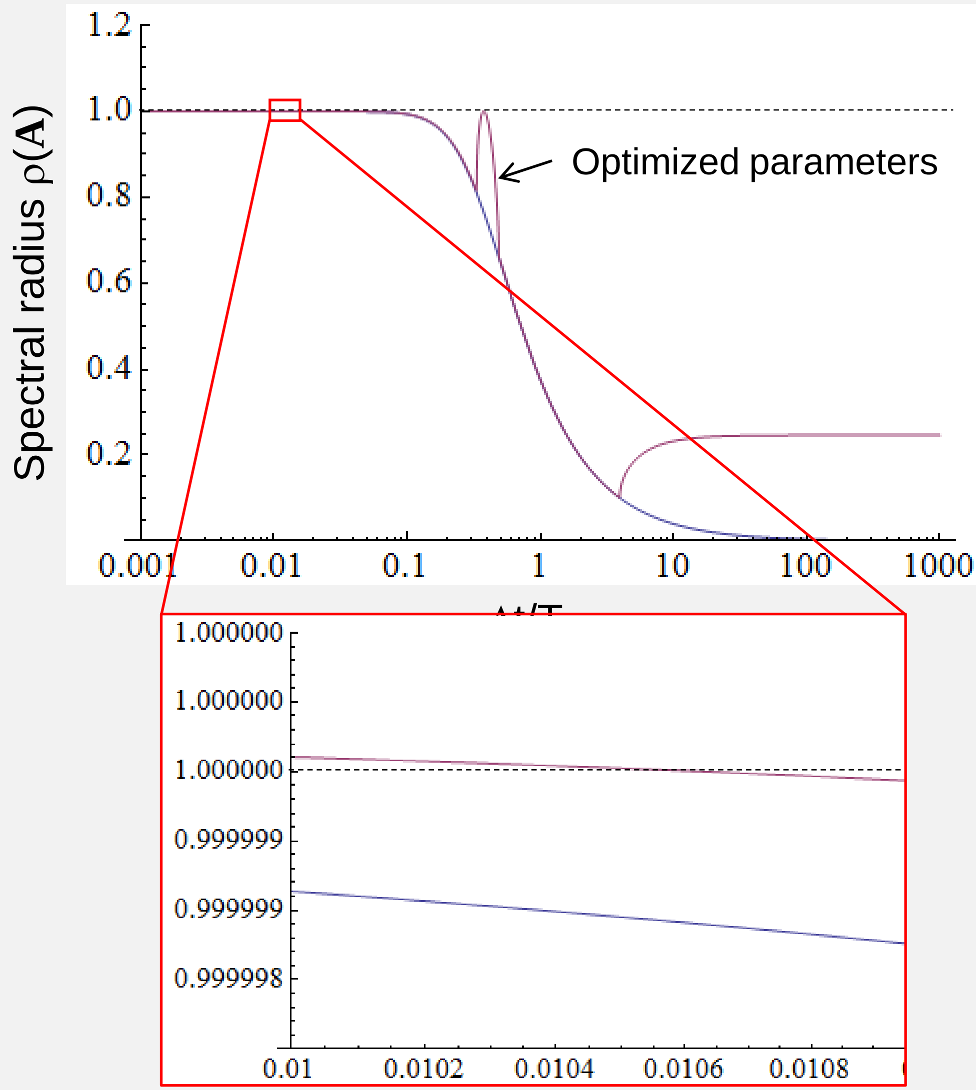
  $for \quad 0 \leq \Delta t/T \leq 1000$

Results

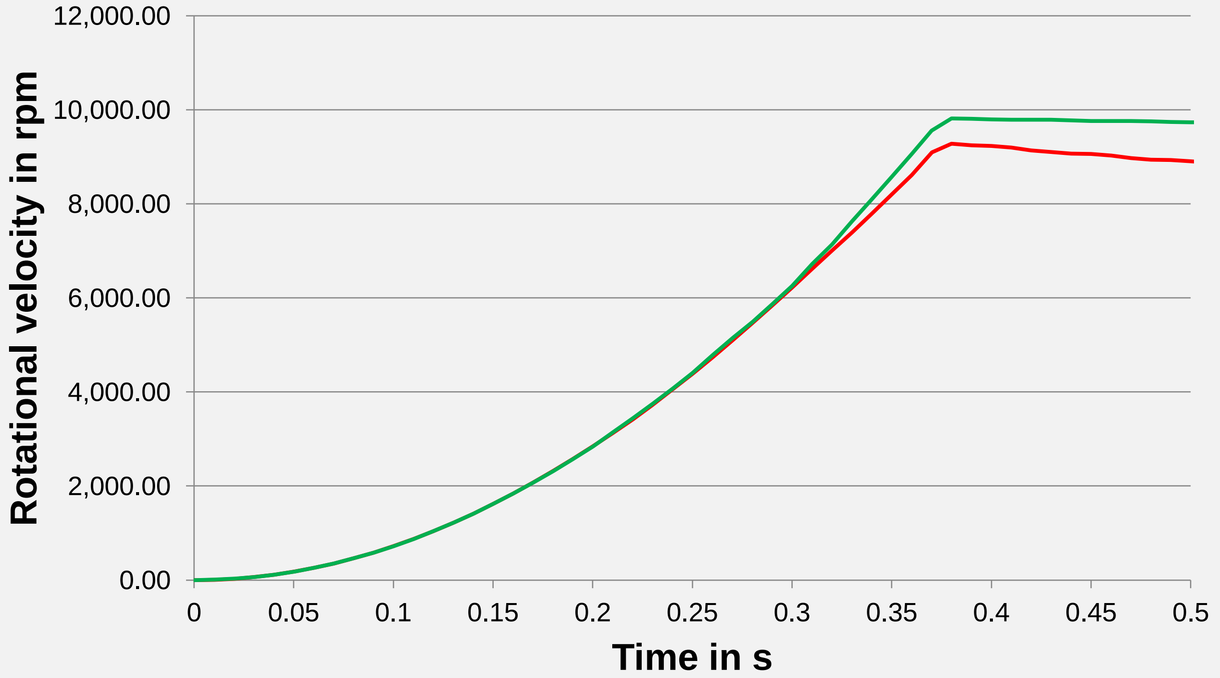
- $\alpha = 0.80767719$

  $\gamma = 0.73496847$

  $\beta = 0.33580917$



Solution with optimized Newmark-Euler parameters

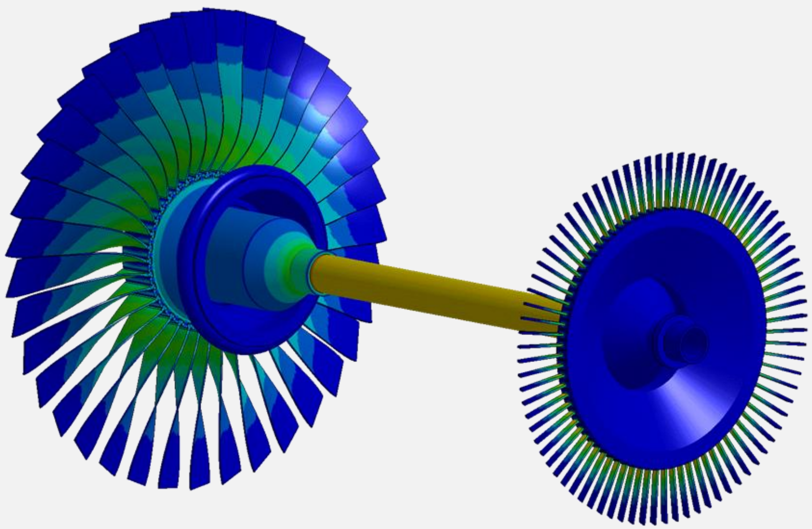


- Solution with optimized parameters stays stable
- Numerical damping in interesting regions is reduced

Summary

- Instability issues of implicit simulation of flexible rotating structures are solved
- Newmark-Euler (Composite) time-integration seems to be advantageous for the simulation of rotating structures compared to classical Newmark time-integration
- Numerical damping of Newmark-Euler scheme can be reduced by optimization of parameters
- Optimized parameters/change of parameters should be handled with care (Computation of stability map very useful)
- Simulations with optimized Newmark-Euler parameter set led to stable results with improved accuracy

Thank you for your attention!



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