EFG and XFEM Cohesive Fracture Analysis Methods in LS-DYNA

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Outline

1. Overview on Failure/Crack Simulations

2. EFG and XFEM Cohesive Fracture Methods

3. Numerical Examples

4. Conclusions

1. Overview on Failure/Crack Simulations

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- **Tie-break Interface**
  Force/stress-based failure + spring element, rigid rods, or other constraints
  Suitable for delamination, debonding, known weak areas

- **Element Erosion**
  Stress/strain-based failure + contact force
  Loss of conservation, strong mesh dependence and inadequate accuracy

- **Cohesive Interface Element**
  Cohesive zone model + interface element + contact force
  Crack along interfaces: Mesh dependence

- **EFG**
  Cohesive zone model + moving least-square + EFG visibility

- **XFEM**
  Cohesive zone model + level sets + extended finite element

EFG and XFEM Failure Analysis

- ❑ Both are discrete approaches (strong discontinuity).
- ❑ Crack initiation and propagation are governed by cohesive law (Energy release rate).
- ❑ Crack propagates cell-by-cell in current implementation.
- ❑ EFG is defined by visibility; XFEM is defined by Level Set.
- ❑ Minimized mesh sensitivity and orientation effects in cracks.
- ❑ Applied to quasibrittle materials and some ductile materials.
- ❑ EFG for Solid with 4-noded integration cells.
- ❑ XFEM for 2D plain strain and shells.

2.1 EFG Cohesive Failure Method

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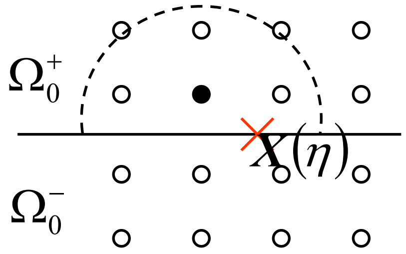
Meshfree Method: MLS + Visibility Criterion (Belytschko *et al.* 1996)

Moving Least Square



$$\mathbf{u}^h(\mathbf{X}) = \sum_{I=1} \Phi_I(\mathbf{X})\mathbf{u}_I$$

$$\Phi_I = \mathbf{P}(\mathbf{X})^T \mathbf{A}(\mathbf{X})^{-1} \mathbf{P}(\mathbf{X}_I) W(\mathbf{X}-\mathbf{X}_I, h)$$

$$\mathbf{A}(\mathbf{X}) = \sum_J \mathbf{P}(\mathbf{X}_J)\mathbf{P}^T(\mathbf{X}_J) W(\mathbf{X}-\mathbf{X}_I, h)$$

Visibility Criterion

Intrinsic (implicit) crack: no additional unknowns

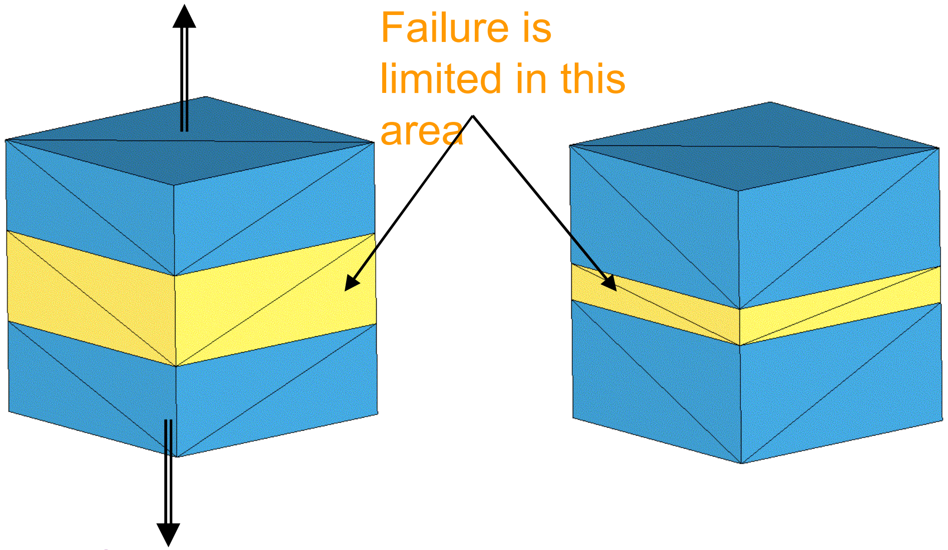
The domain of influence of particles on one side of the crack cannot go through the crack surface and the particles on one side of the crack cannot interact with the particles on other side of the crack

Mid-plane fracture surface

$$\mathbf{x}(\eta) = \sum_{I=1}^{2} \Phi_I^{FEM}(\eta)\mathbf{X}_I + \frac{1}{2}\left(\sum_{J\in\Omega_0^+} \Psi_J(\mathbf{X}(\eta))\mathbf{u}_J + \sum_{J\in\Omega_0^-} \Psi_J(\mathbf{X}(\eta))\mathbf{u}_J \right)$$

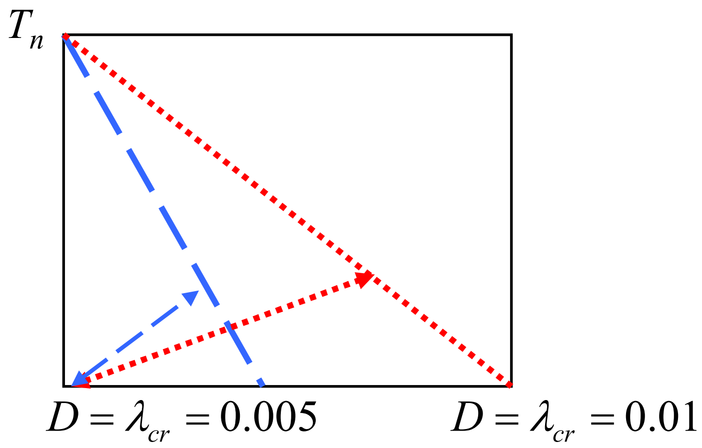
$$\frac{\partial \mathbf{x}(\eta)}{\partial \eta} = \sum_{I=1}^{2} \mathbf{X}_I \otimes \frac{\partial \Phi_I^{FEM}(\eta)}{\partial \eta} + \frac{1}{2}\left(\sum_{J\in\Omega_0^+} \mathbf{u}_J \otimes \frac{\partial \Psi_J(\mathbf{X})}{\partial \mathbf{X}} + \sum_{J\in\Omega_0^-} \mathbf{u}_J \otimes \frac{\partial \Psi_J(\mathbf{X})}{\partial \mathbf{X}} \right) \frac{\partial \mathbf{X}(\eta)}{\partial \eta}$$

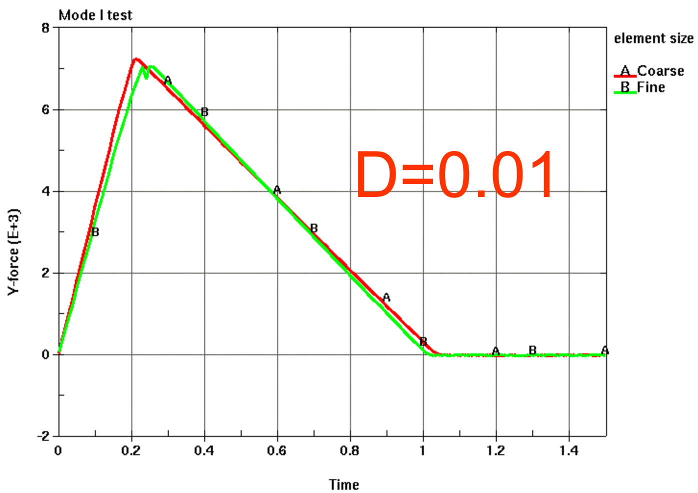
Minimization of Mesh Size Effect in Mode-I Failure Test

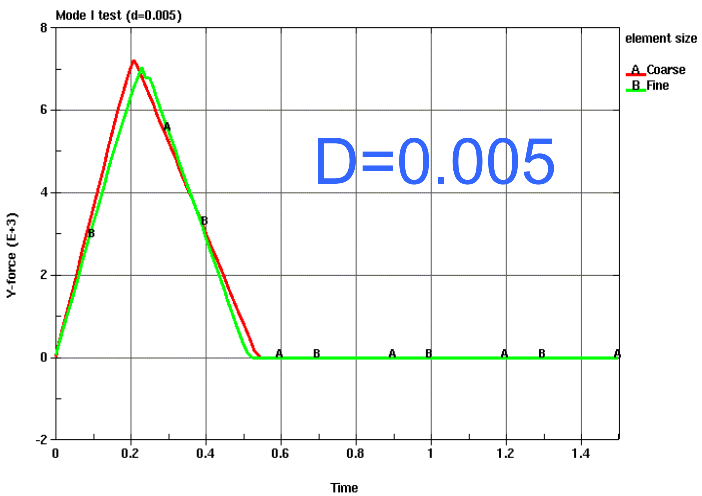


Coarse elements

Fine elements







Input Format for EFG Failure Analysis

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*SECTION_SOLID_EFG

Card 2

Variable	DX	DY	DZ	ISPLINE	IDILA	IEBT	IDIM	TOLDEF
Type	F	F	F	I	I	I	I	F
Default	1.01	1.01	1.01	0	0	-1	2	0.01

IDIM EQ. 1: Local boundary condition method
EQ. 2: Two-points Guass integration (default)
EQ.-1: Stabilized EFG method (applied to 8-noded, 6-noded and combination of them)
EQ.-2: Fractured EFG method (applied to 4-noded & SMP only)

Card 3

Variable	IGL	STIME	IKEN	SF	CMID	IBR	DS	ECUT
Type	I	F	I	F	I	I	F	F
Default	0	1.e+20	0	0.0		1	1.01	0.1

SF: Failure strain
CMID: Cohesive material ID
IBR: Branching indicator
DS: Normalized support for displacement jump
ECUT: Minimum edge cut

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*SECTION_SOLID_EFG
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1.1, 1.1, 1.1, , ,4, -2,
, , , , 100, 1, 2.0, 0.2
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100,1.0e-07, ,1, 330.0, 0.0001,
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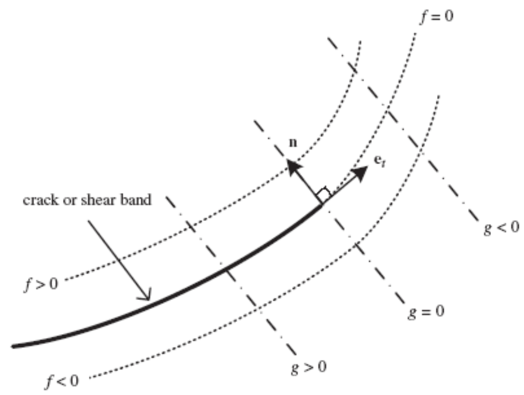
Extended FEM: Level Set + Local PU (Belytschko *et al*. 2000)

Level Set

Discontinuity defined by two implicit functions: $f(\mathbf{X})$ and $g(\mathbf{X})$

Signed distance function $f(\mathbf{X}) = \min_{\bar{\mathbf{X}}\in\Gamma_\alpha} \|\mathbf{X}-\bar{\mathbf{X}}\| sign[\mathbf{n}\cdot(\mathbf{X}-\bar{\mathbf{X}})]$

Discontinuity $\mathbf{X}\in\Gamma_\alpha^0$ if $f(\mathbf{X})=0$ and $g(\mathbf{X},t)>0$



Define implicit functions locally

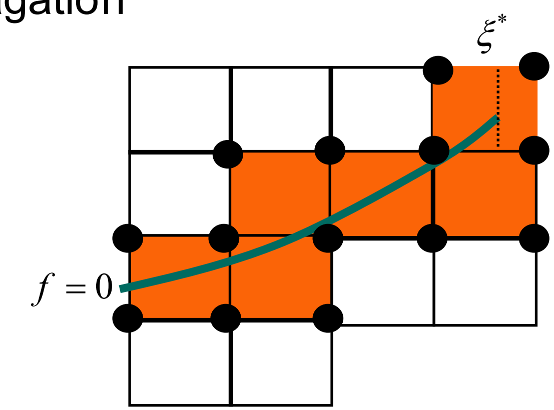
$$f(\mathbf{X}) = \sum_I f_I N_I(\mathbf{X})$$

$g(\mathbf{X},t)$ replaced by index for elementwise crack propagation

Local Partition of Unity

$$\mathbf{u}^h(\mathbf{X}) = \sum_{I=1} \Phi_I^{FEM}(\xi)\mathbf{u}_I + \sum_{I\in w} \Psi_I(\mathbf{X})\mathbf{q}_I$$

$$\Psi_I(\mathbf{X}) = \begin{cases} \Phi_I^{FEM}(\xi)[H(f(\mathbf{X}))-H(f(\mathbf{X}_I))] & \text{fully cut element} \\ \Phi_I^{FEM}(\xi^*)[H(f(\mathbf{X}))-H(f(\mathbf{X}_I))] & \text{contain crack tip} \end{cases}$$



Phantom Nodes and Phantom Elements

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Approximation of crack in element - Song, Areias and Belytschko (2006)

$$\mathbf{u}^h(\mathbf{X},t) = \sum_I N_I(\mathbf{X})\{\mathbf{u}_I(t) + \mathbf{q}_I(t)[H(f(\mathbf{X})) - H(f(\mathbf{X}_I))]\}$$

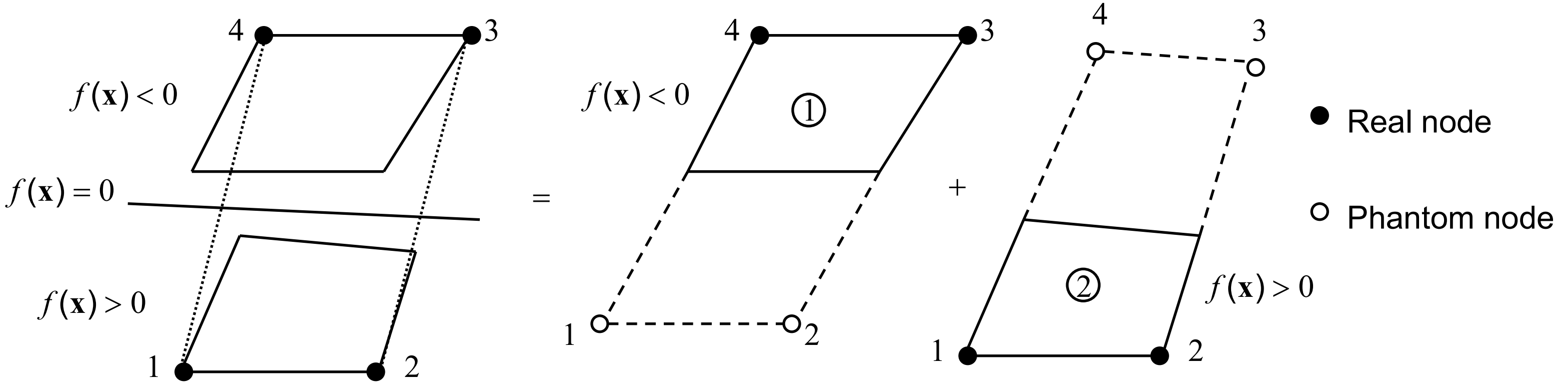
Rewrite into superposition of two phantom elements

$$\mathbf{u}^h(\mathbf{X},t) = \sum_{I\in S_1} \underbrace{\mathbf{u}_I^1(t)N_I(\mathbf{X})}_{\mathbf{u}^1(\mathbf{X},t)} H(-f(\mathbf{X})) + \sum_{I\in S_2} \underbrace{\mathbf{u}_I^2(t)N_I(\mathbf{X})}_{\mathbf{u}^2(\mathbf{X},t)} H(f(\mathbf{X}))$$

where

$$\mathbf{u}_I^1 = \begin{cases} \mathbf{u}_I & \text{if } f(\mathbf{X}_I) < 0 \\ \mathbf{u}_I - \mathbf{q}_I & \text{if } f(\mathbf{X}_I) > 0 \end{cases}$$

$$\mathbf{u}_I^2 = \begin{cases} \mathbf{u}_I + \mathbf{q}_I & \text{if } f(\mathbf{X}_I) < 0 \\ \mathbf{u}_I & \text{if } f(\mathbf{X}_I) > 0 \end{cases}$$



Domain Integration Schemes

Phantom Element Integration

- Song, Areias and Belytschko (2006)

Integration in phantom elements and assembly according to area ratios

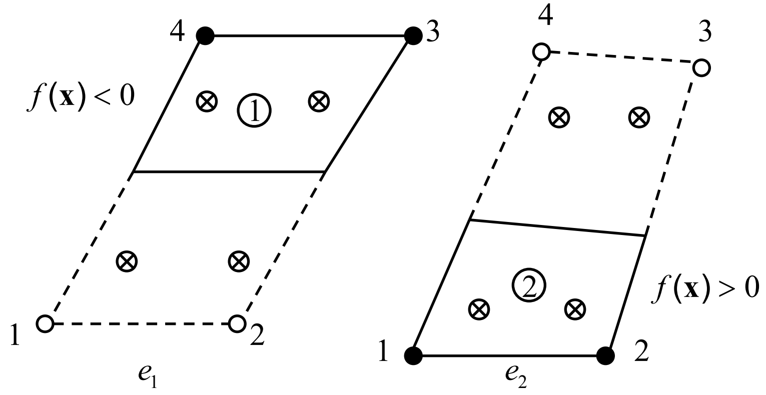
$$\mathbf{f}^{kin}_{(e_1/e_2)} = \frac{A_{(e_1/e_2)}}{A_0}\int_{\Omega_0^e} \rho_0 \mathbf{N}^T\mathbf{N}\, d\Omega_0^e\, \ddot{\mathbf{u}}_{(e_1/e_2)}$$

$$\mathbf{f}^{int}_{(e_1/e_2)} = \frac{A_{(e_1/e_2)}}{A_0}\int_{\Omega_0^e} \mathbf{B}^T\mathbf{P}\, d\Omega_0^e$$

$$\mathbf{f}^{ext}_{e_1} = \frac{A_{e_1}}{A_0}\int_{\Omega_0^e} \rho_0 \mathbf{N}^T\mathbf{b}\, d\Omega_0^e + \int_{\Gamma_{0,t}^e} \mathbf{N}^T\mathbf{t}^0 H[-f(\mathbf{X})]\, d\Gamma_{0,t}^e$$

$$\mathbf{f}^{ext}_{e_2} = \frac{A_{e_2}}{A_0}\int_{\Omega_0^e} \rho_0 \mathbf{N}^T\mathbf{b}\, d\Omega_0^e + \int_{\Gamma_{0,t}^e} \mathbf{N}^T\mathbf{t}^0 H[f(\mathbf{X})]\, d\Gamma_{0,t}^e$$

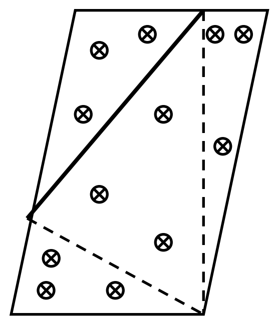
$$\mathbf{f}^{coh}_{e_1} = -\int_{\Gamma_{0,c}^e} \mathbf{N}^T\boldsymbol{\tau}^c\mathbf{n}_0\, d\Gamma_{0,c}^e \qquad \mathbf{f}^{coh}_{e_2} = \int_{\Gamma_{0,c}^e} \mathbf{N}^T\boldsymbol{\tau}^c\mathbf{n}_0\, d\Gamma_{0,c}^e$$



Easier in implementation

Sub-domain integration

Integration conducted in two sub-domains cut by cracks



More accurate results

Difficulties in implementation: Varied integration schemes, Different data structure, Transfer of state variables

*SECTION_SHELL{_XFEM}

Card 1

Variable	SECID	ELFORM	SHRF	NIP	PROPT	QR/IRID	ICOMP	SETYP
Type	I	I	F	I	F	F	I	I

ELFORM EQ. 52: Plane strain (x-y plane) XFEM
EQ. 54: Shell XFEM

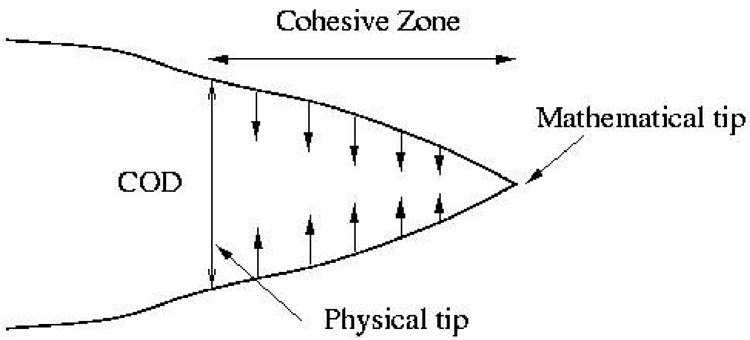
Card 3

Variable	CMID	IOPBASE	IDIM	INITC				
Type	I	I	I	I				
Default		13,16	0	1				

CMID: Cohesive material ID
IOPBASE: Base element type
Type 13 for plain strain XFEM
Type 16 for shell XFEM
IDIM: Domain integration method
0 for phantom element integration
1 for subdomain integration
INITC: Criterion for crack initiation
1 for maximum tensile stress

```
*SECTION_SHELL
5, 53
0.1, 0.1, 0.1, 0.1
100, 16, 0, 1
*MAT_COHESIVE_TH
100,1.0e-07, ,1, 330.0, 0.0001,
```

2.3 Cohesive Fracture Model



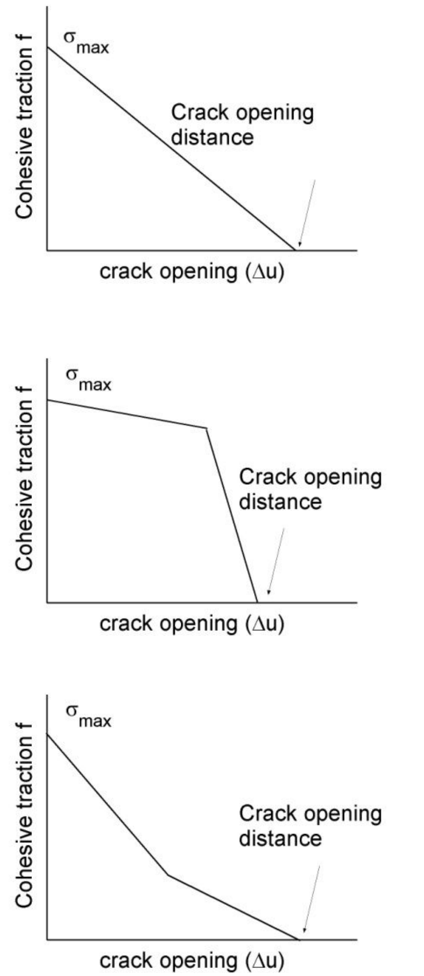
❖ Crack is consisted of mathematical crack (cohesive zone) and physical crack.

❖ Cohesive zone crack initiates when maximum stress reached.
Physical crack occurs when critical COD reached.
Cohesive work = critical energy release rate

❖ Constitutive cohesive law relates the traction forces to displacement jumps through a potential:

$$\mathbf{T} = \frac{\partial \Phi(\boldsymbol{\delta}, \mathbf{q})}{\partial \boldsymbol{\delta}}$$

❖ Displacement jumps can have various components due to different crack modes.



Different Cohesive Laws

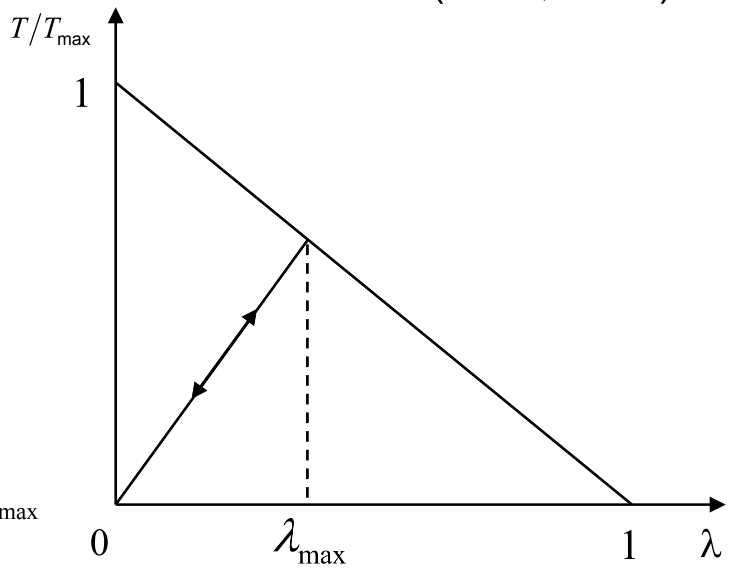
Constitutive Cohesive Law

Effective displacement jump

- Zavattieri (2001, 2005)

$$\lambda = \sqrt{\left(\frac{u_n}{\delta_n}\right)^2 + \beta_1^2\left(\frac{u_{t1}}{\delta_{t1}}\right)^2 + \beta_2^2\left(\frac{u_{t2}}{\delta_{t2}}\right)^2 + \hat{\beta}^2\left(\frac{\Delta\theta}{\Delta\theta_{\max}}\right)^2}$$

$$G_{Ic} = \frac{1}{2}\delta_n T_{\max}, \quad \frac{G_{IIc}}{G_{Ic}} = \beta_1, \quad \frac{G_{IIIc}}{G_{Ic}} = \beta_2, \quad \hat{\beta} = \sqrt{\frac{\Delta\theta_{\max} t}{6\delta_n}}$$



Initially Rigid Cohesive Law

Tractions

$$T_n = \frac{\partial\Phi}{\partial u_n} = \frac{1-\lambda}{\lambda}\left(\frac{u_n}{\delta_n}\right)T_{\max}$$

$$\lambda = \max(\lambda_{\max}, \lambda)$$

$$\lambda_{\max} = 0, \quad \lambda_{\max} = \lambda \text{ if } \lambda > \lambda_{\max}$$

$$T_{t1} = \frac{\partial\Phi}{\partial u_{t1}} = \frac{1-\lambda}{\lambda}\left(\frac{u_{t1}}{\delta_{t1}}\right)\alpha_1 T_{\max}$$

$$T_{t2} = \frac{\partial\Phi}{\partial u_{t2}} = \frac{1-\lambda}{\lambda}\left(\frac{u_{t2}}{\delta_{t2}}\right)\alpha_2 T_{\max}$$

$$M_{t1} = \frac{\partial\Phi}{\partial\Delta\theta} = \frac{1-\lambda}{\lambda}\left(\frac{\Delta\theta}{\Delta\theta_{\max}}\right)\hat{\alpha} T_{\max}$$

$$\alpha_1 = \beta_1^2\left(\frac{\delta_n}{\delta_{t1}}\right), \quad \alpha_2 = \beta_2^2\left(\frac{\delta_n}{\delta_{t2}}\right), \quad \hat{\alpha} = \hat{\beta}^2\frac{\delta_n}{\Delta\theta_{\max}}$$

Equivalent fracture stress

$$T_{efs} \equiv \sqrt{T_n^2 + \left(\frac{\beta_1}{\alpha_1}\right)^2 T_{t1}^2 + \left(\frac{\beta_2}{\alpha_2}\right)^2 T_{t2}^2 + \left(\frac{\hat{\beta}}{\hat{\alpha}}\right)^2 M_{t1}^2} \geq T_{\max}$$

2.4 Computation Procedures

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$$\delta W^{kin} = \delta W^{\text{int}} - \delta W^{ext} + \delta W^{coh} \quad \forall\, \delta u(X) \in u_0$$

$$\delta W^{kin} = \int_{\Omega_0} \delta \boldsymbol{u} \cdot \rho_0 \ddot{\boldsymbol{u}} d\Omega_0$$

$$\delta W^{\text{int}} = \int_{\Omega_0} \frac{\partial \delta \boldsymbol{u}}{\partial \boldsymbol{X}} : \boldsymbol{P} d\Omega_0$$

$$\delta W^{ext} = \int_{\Omega_0} \delta \boldsymbol{u} \cdot \rho_0 \boldsymbol{b} d\Omega_0 + \int_{\Gamma_t^0} \delta \boldsymbol{u} \cdot \bar{\boldsymbol{t}}^0 d\Gamma_t^0$$

$$\delta W^{coh} = -\int_{\Gamma_c} \delta [[\boldsymbol{u}]] \cdot \boldsymbol{\tau}^c d\Gamma_c$$

$$\boldsymbol{f}^{kin} = \boldsymbol{f}^{\text{int}} - \boldsymbol{f}^{ext} + \boldsymbol{f}^{coh}$$

$$\boldsymbol{f}_e^{kin} = \int_{\Omega_0^e} \rho_0 \boldsymbol{N}^{\boldsymbol{T}} \boldsymbol{N} H((-1)^e f(\boldsymbol{X})) d\Omega_0^e \ddot{\boldsymbol{u}}$$

$$\boldsymbol{f}_e^{int} = \int_{\Omega_0^e} \boldsymbol{B}^{\boldsymbol{T}} \sigma H((-1)^e f(\boldsymbol{X})) d\Omega_0^e$$

$$\boldsymbol{f}_e^{ext} = \int_{\Omega_0^e} \rho_0 \boldsymbol{N}^{\boldsymbol{T}} \boldsymbol{b} H((-1)^e f(\boldsymbol{X})) d\Omega_0^e + \int_{\Gamma_{0,t}^e} \boldsymbol{N}^{\boldsymbol{T}} \boldsymbol{t} H((-1)^e f(\boldsymbol{X})) d\Gamma_{0,t}^e$$

$$\boldsymbol{f}_e^{coh} = (-1)^e \int_{\Gamma_{0,t}^e} \boldsymbol{N}^{\boldsymbol{T}} \tau^c \boldsymbol{n}_0 d\Gamma_{0,t}^e$$

Cohesive tractions treated as external forces

1. Representation of Cracks

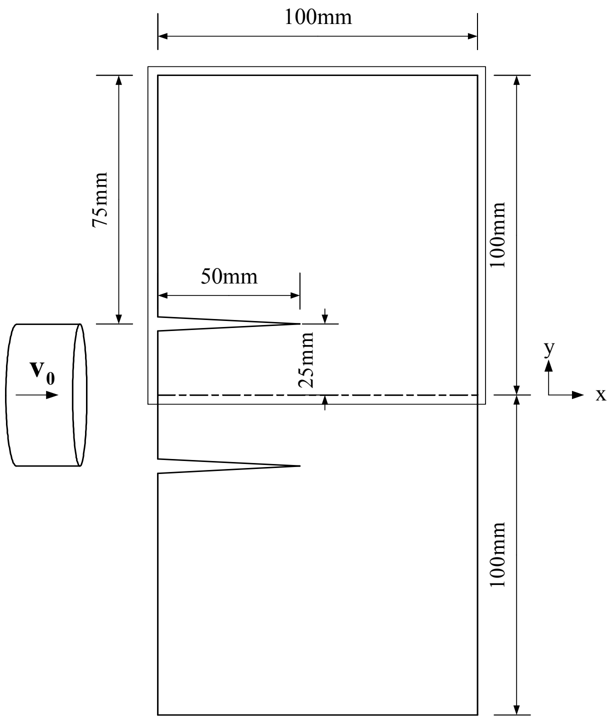
2. Cohesive Law
Crack initiation/propagation

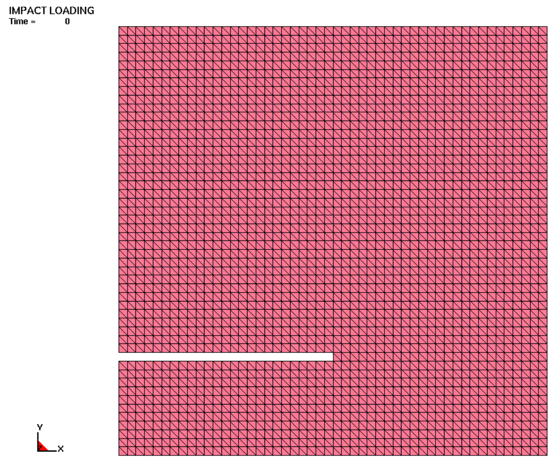
3. Branching/Multiple cracks

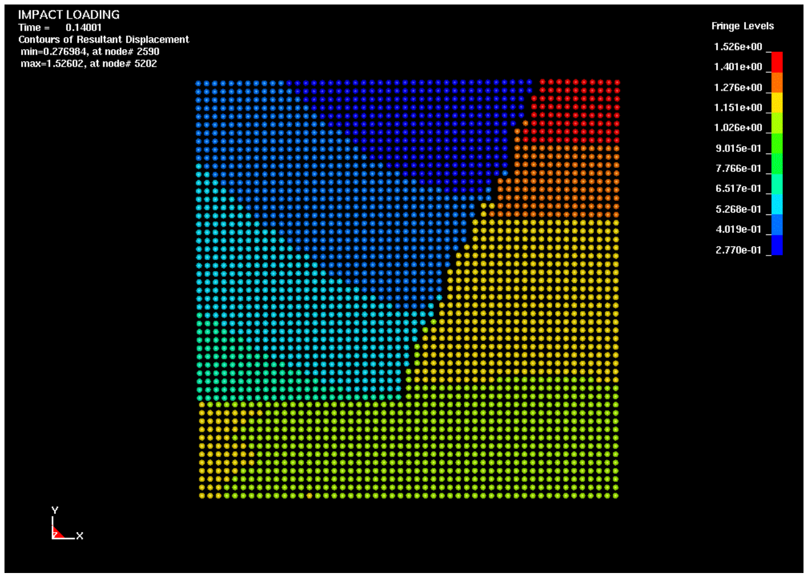
4. State Variables Transfer

5. Numerical Integration

3.1 Kalthoff Plate Crack Propagation







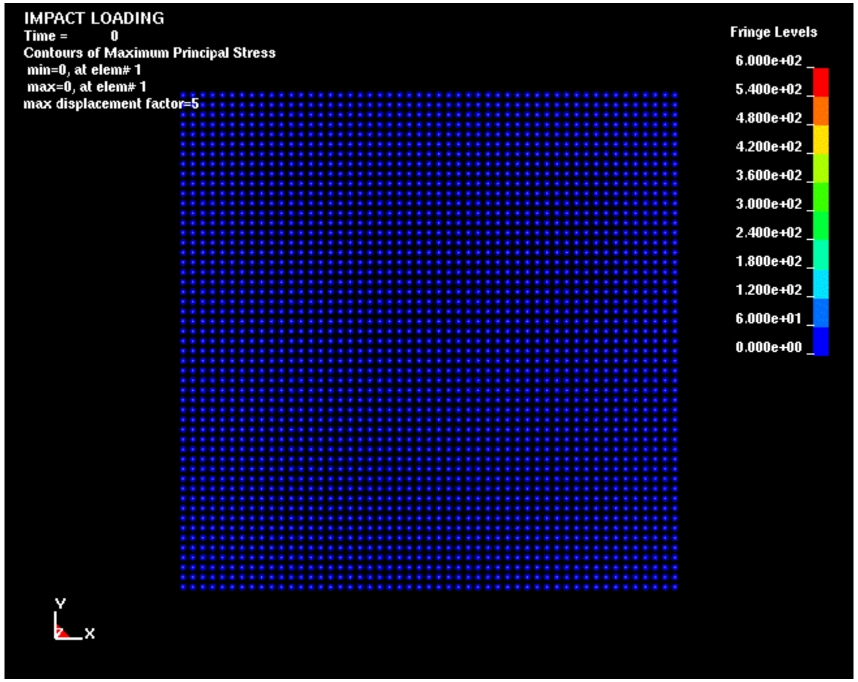
$$\rho = 8000\,\mathrm{kg/m^3}, \quad \boldsymbol{E} = 190\,\mathrm{GPa}, \quad \nu = 0.30$$

$$\boldsymbol{G}_{\boldsymbol{Ic}} = 1.213 \times 10^4\ \mathrm{N/m}, \quad \delta_{\max} = 5.245 \times 10^{-5}\ \mathrm{m}$$

$$\mathbf{v}_0 = 16.5\,\mathrm{m/s}$$

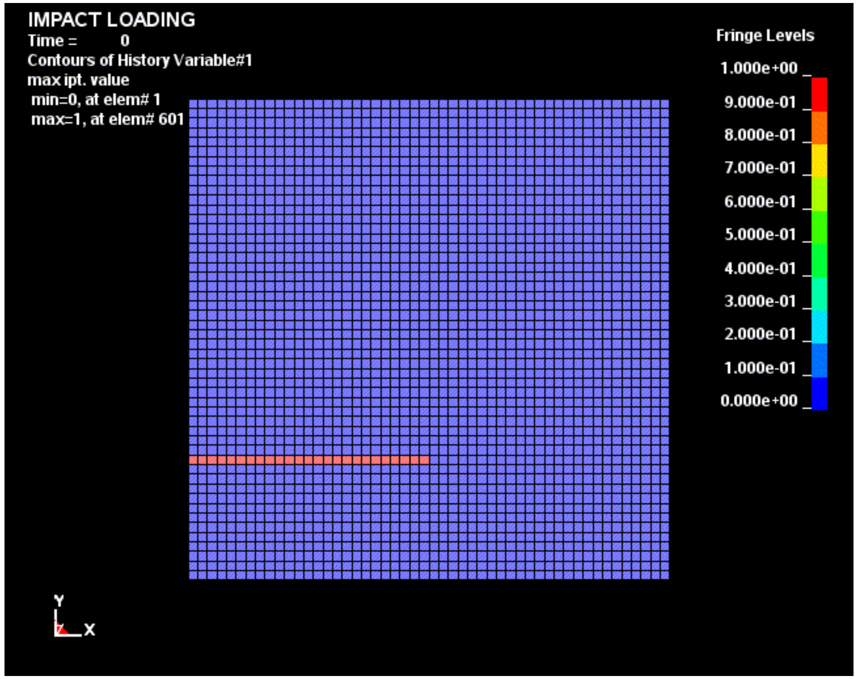
Kalthoff Plate Crack Propagation

EFG 3D
Maximum Principle Stress Contour



Average Crack Angle: 69.0°

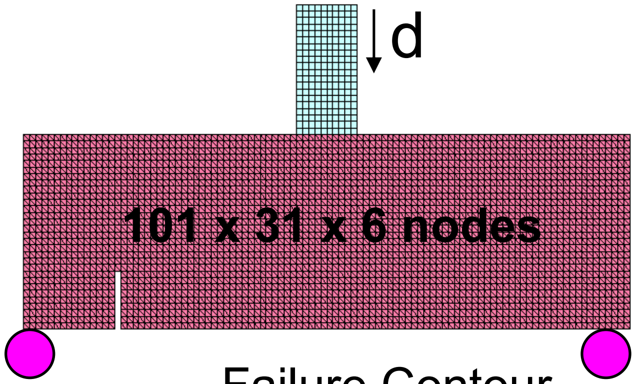
XFEM Plain Strain
Failure Indicator



Average Crack Angle: 62.5°

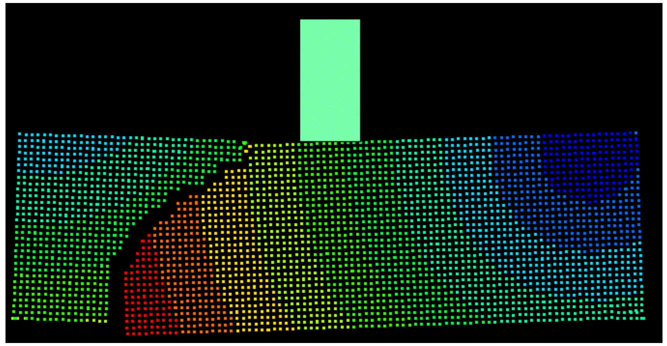
Average Crack Angle from Experiment: 70.0°

3.2 EFG 3D Edge-cracked Plate under Three-point Bending

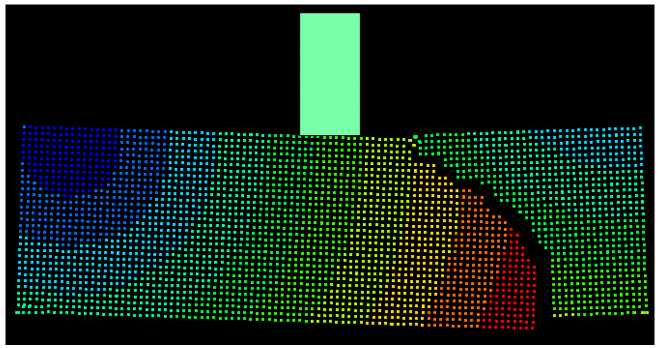


Elastic
EFG Fracture
Linear Cohesive Law
Explicit analysis

front

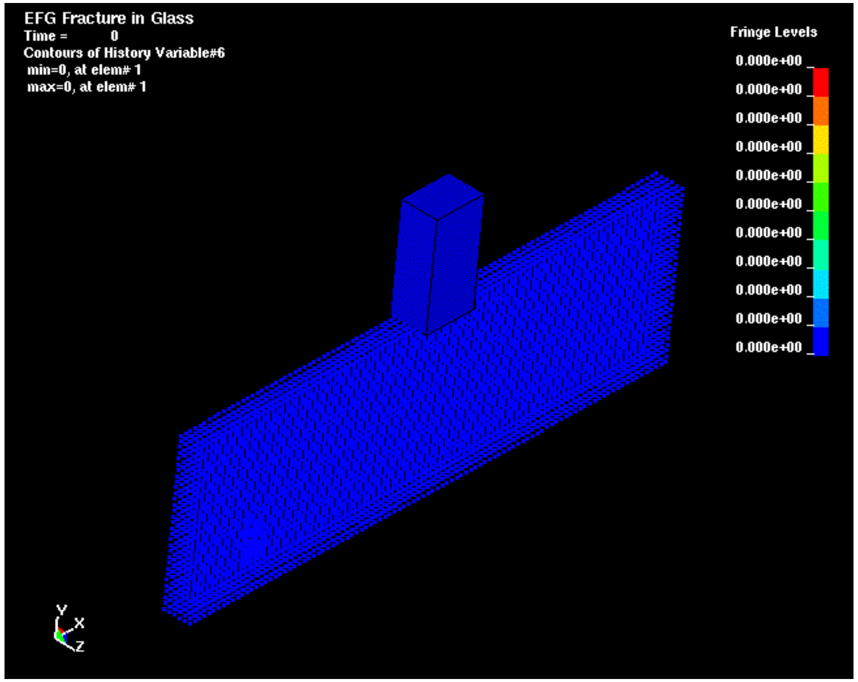


Resultant Displacement Contour



back

Failure Contour



3.3 Rigid Ball Impact on EFG Concrete Plate

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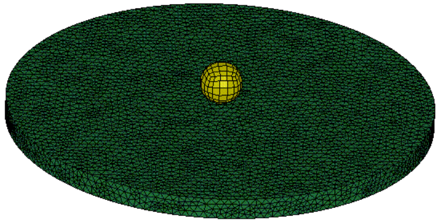
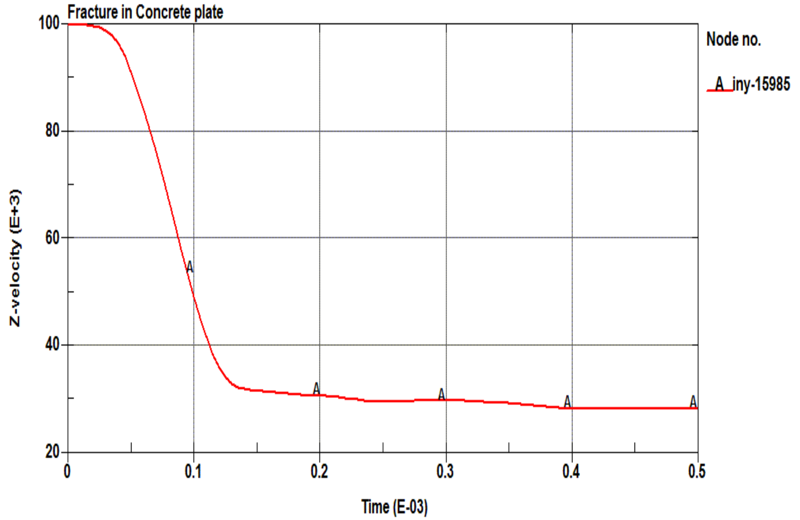
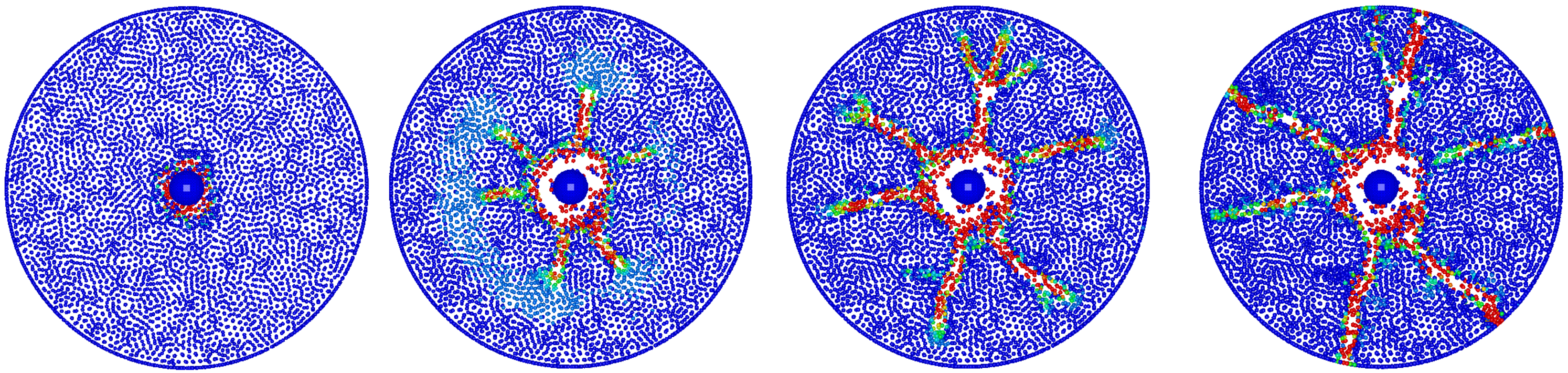


Time-velocity of the rigid ball

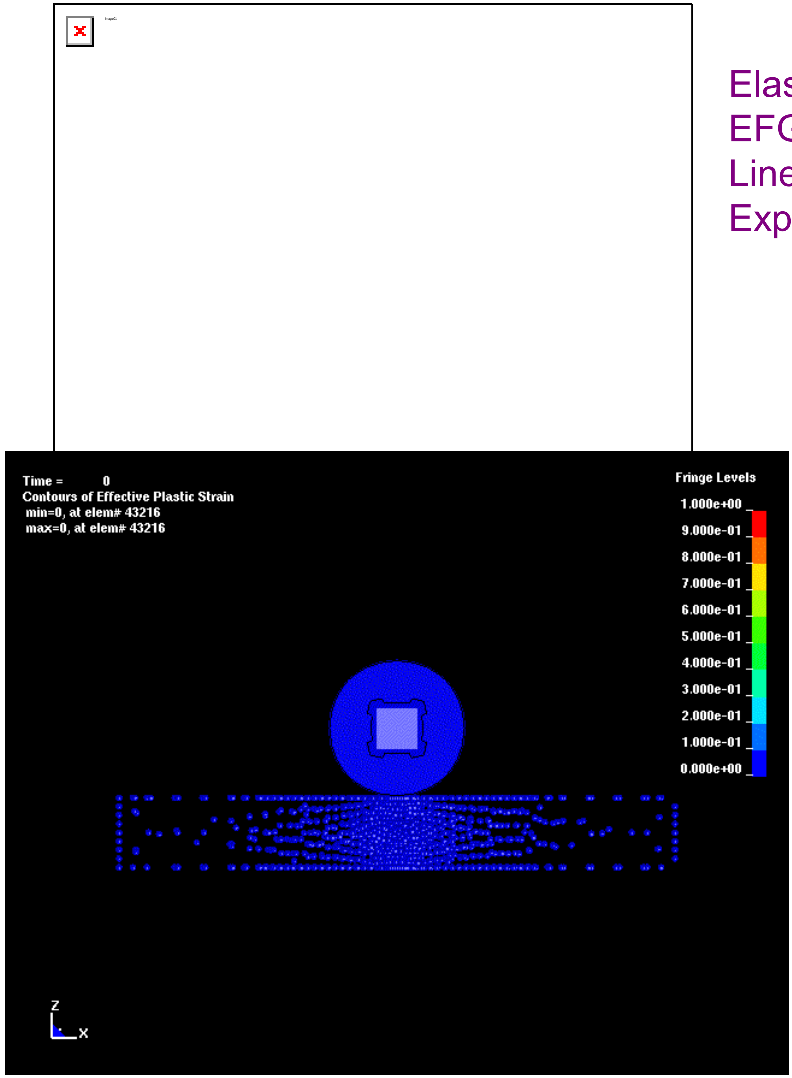


Elastic
EFG Fracture
Linear Cohesive Law
Explicit analysis

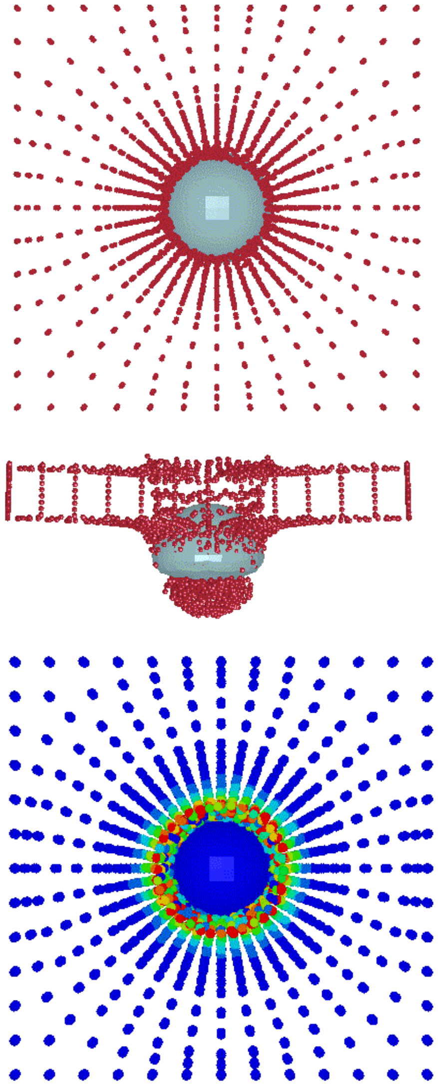


Progressive Crack Propagation

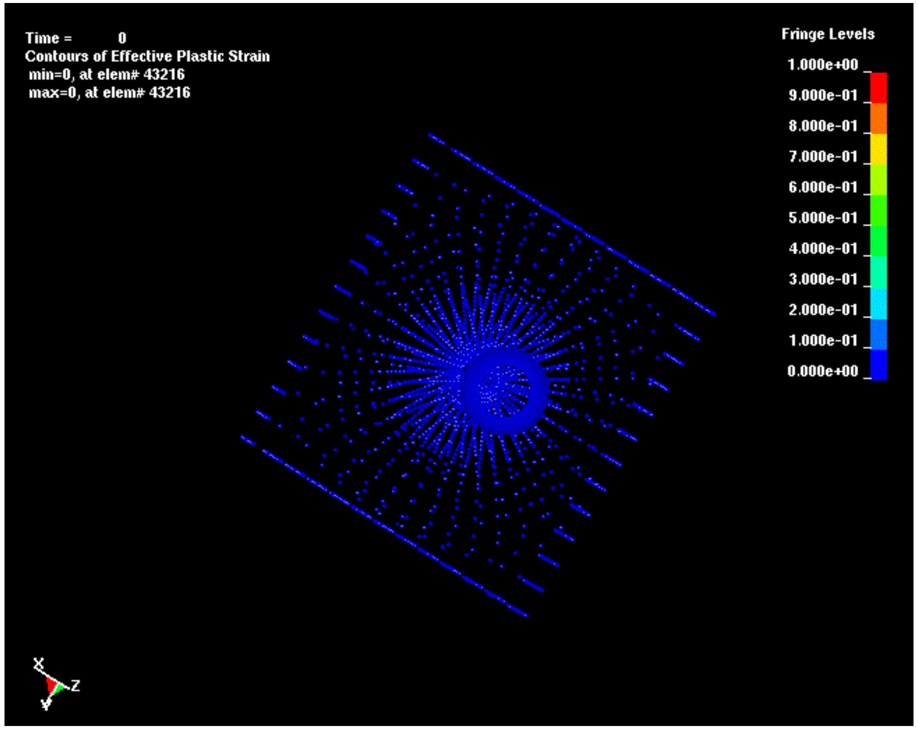
3.4 Steel Ball Impact on Steel Plate



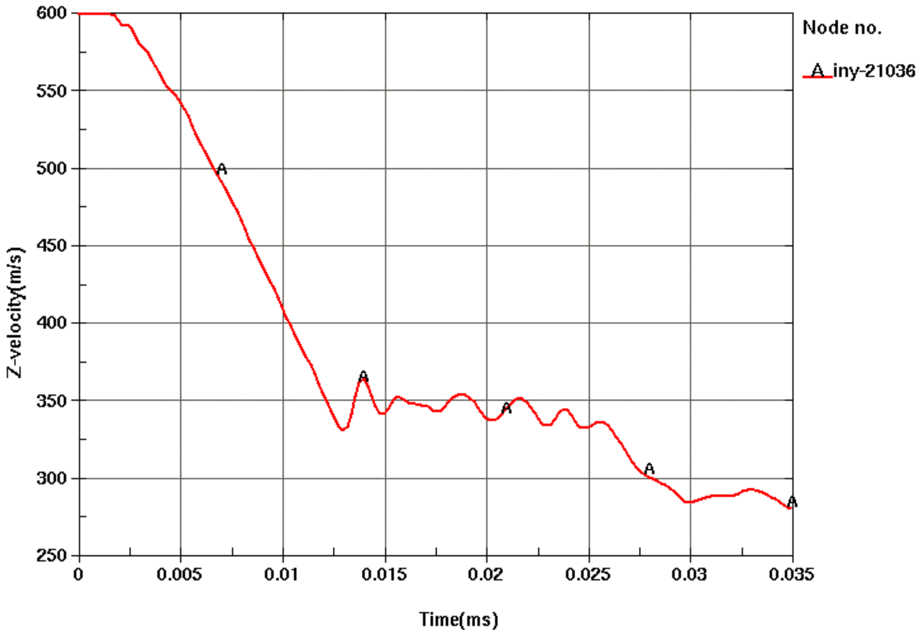
Elastic_plastic
EFG Fracture
Linear Cohesive Law
Explicit analysis



Steel Ball Impact on Steel Plate

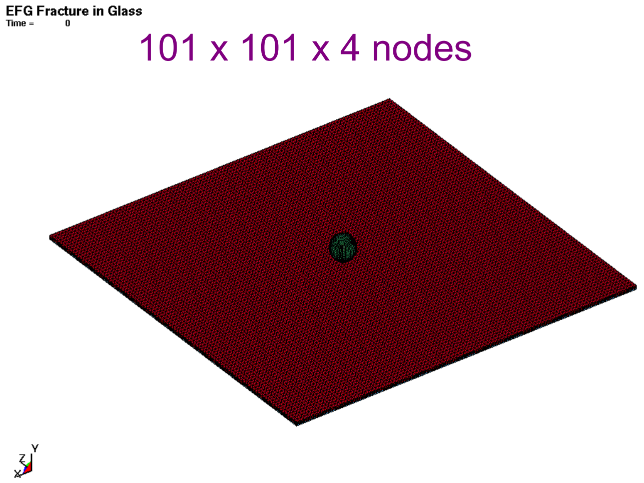


Time-velocity of the metal ball



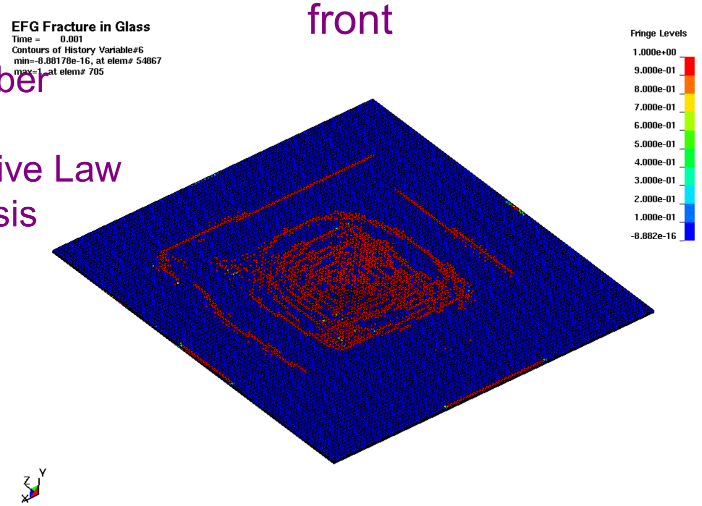
3.5 EFG Glass under Impact

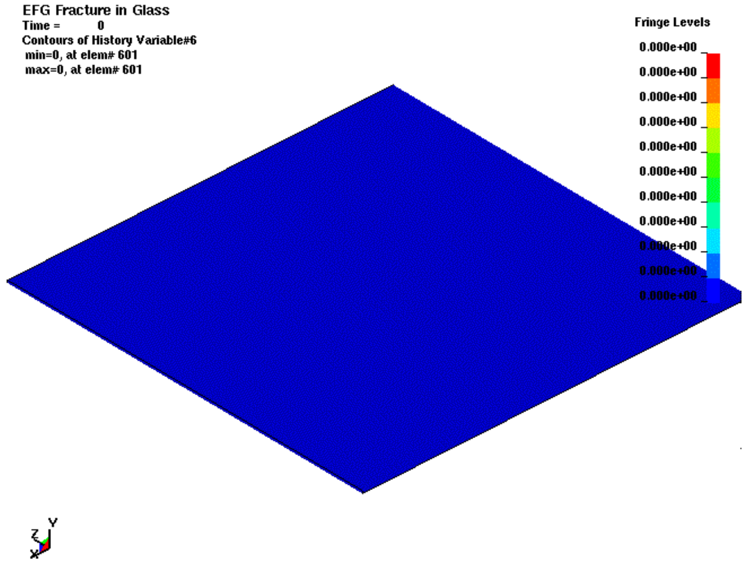
101 x 101 x 4 nodes



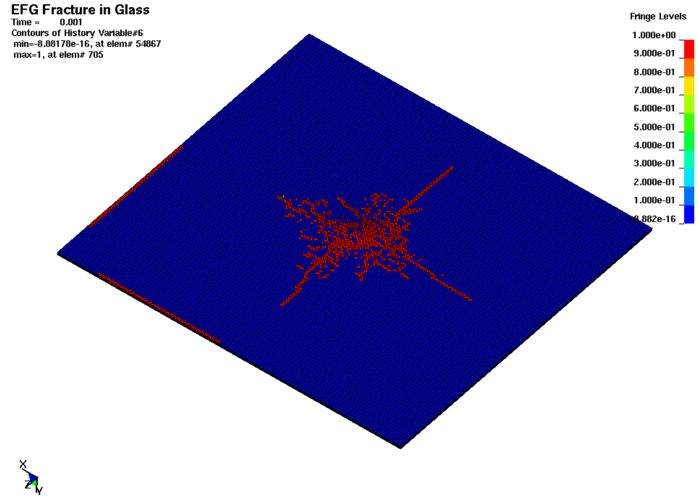
Elastic + Rubber
EFG Fracture
Linear Cohesive Law
Explicit analysis

front



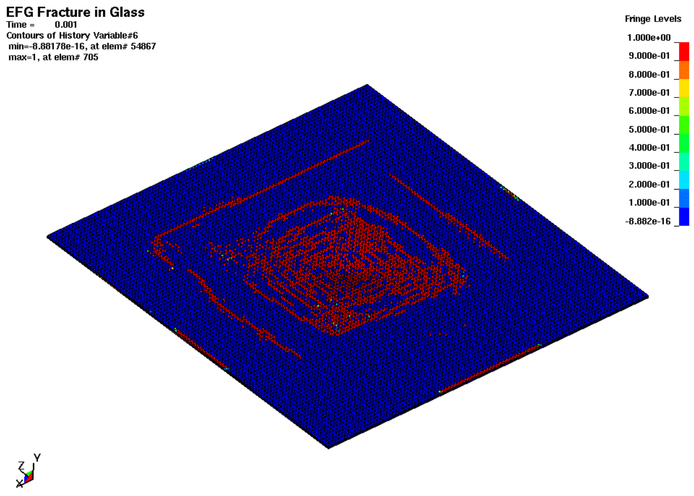


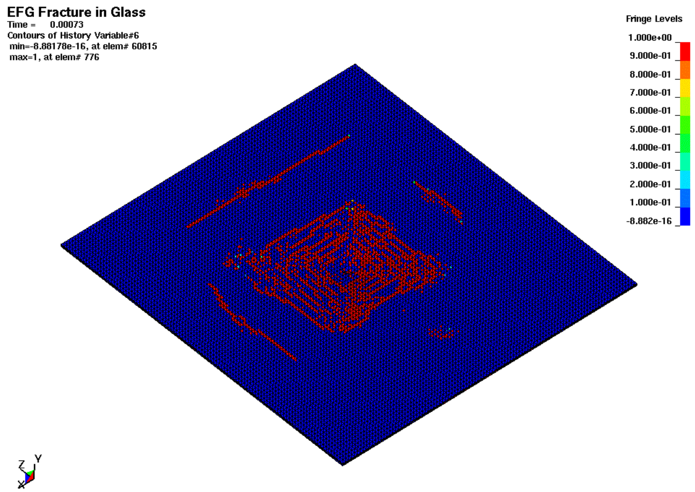
back

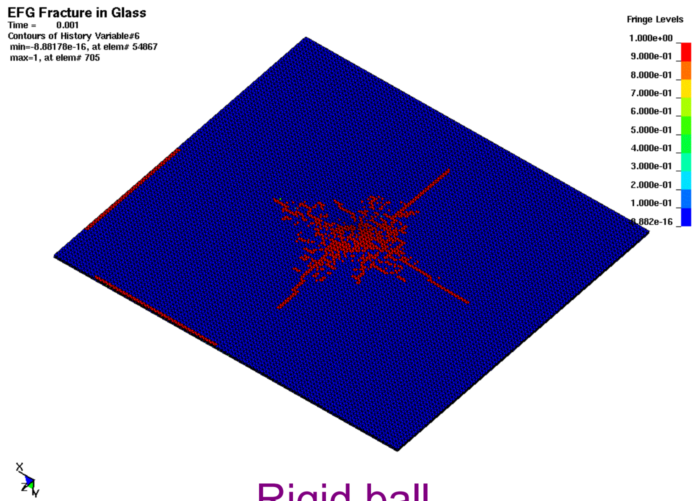


Failure Contour

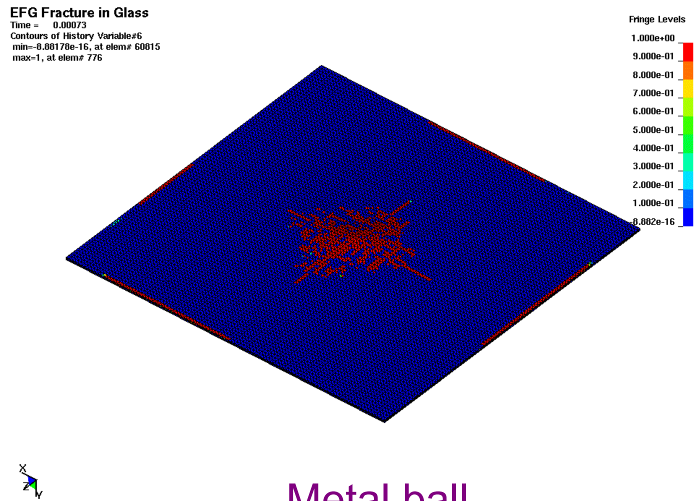
3.5 EFG Glass under Impact





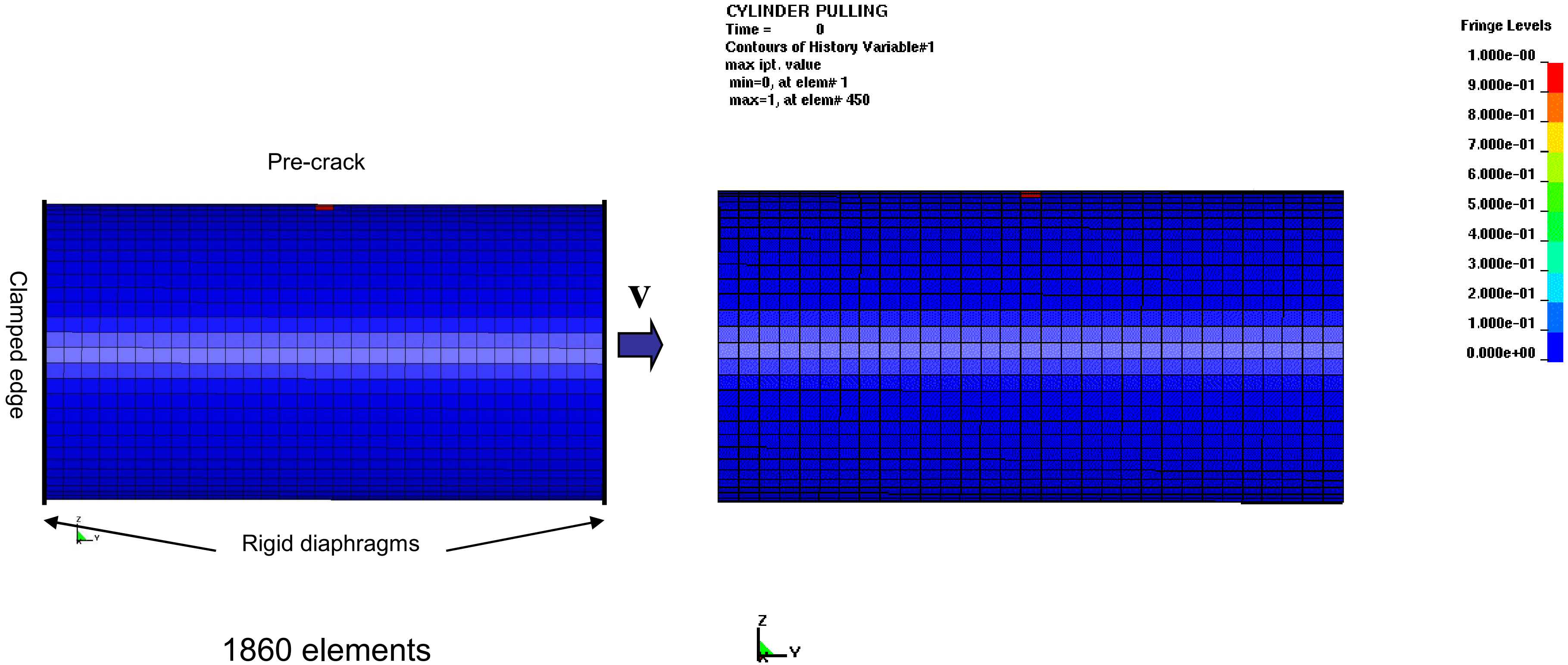


Rigid ball



Metal ball

3.6 Thin Cylinder Pulling



1860 elements

$$\rho = 7830\,\text{kg/m}^3, \quad E = 70\,\text{GPa}, \quad \nu = 0.3, \quad \sigma_y = (100 + 200\varepsilon^p)\text{MPa}$$

$$G_{Ic} = 500\text{kN/m}, \quad T_{\text{max}} = 200\text{MPa}$$

$$R = 50\text{mm}, \quad L = 200\text{mm}, \quad h = 5\text{mm}$$

$$\mathbf{v} = 5\text{m/s}$$

4. Conclusions

- ❑ EFG and XFEM cohesive failure methods are successfully applied to brittle and semi-brittle materials.
- ❑ EFG failure analysis with visibility criterion is more robust and capable of handling crack branching and interaction.
- ❑ XFEM cohesive failure analysis is more suitable for crack analysis with pre-cracks and without crack branching or interaction.
- ❑ Further research is needed for ductile fracture analysis.