Fuzzy analysis as alternative to stochastic methods – a comparison by means of a crash analysis

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Abstract:

A realistic and reliable numerical simulation demands suitable computational models and applicable data models for the structural design parameters. Structural design parameters are in general non-deterministic. The choice of an appropriate uncertainty model for describing selected structural design parameters depends on the characteristics of the available information. Besides the most often used probabilistic models and the stochastic analysis techniques newer uncertainty models have been developed that offer the chance to take account of non-stochastic uncertainty that frequently appears in engineering problems. In this paper a crash analysis example with uncertain structural parameters is presented. The uncertainty quantification is realized with aid of the uncertainty models randomness and fuzziness. The quantified uncertain structural parameters are introduced into their respective analysis algorithms: the stochastic structural analysis and the fuzzy structural analysis. Specifies and advantages of the uncertainty models fuzziness and randomness and of the associated simulation techniques are addressed.

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Keywords:

fuzziness, randomness, fuzzy structural analysis, stochastic structural analysis, LS-OPT, LS-DYNA, response surface method (RSM)

1 Introduction

Structural engineering mainly focuses on computing structural responses, assessing structural safety, and determining parameters for structural design that meet all relevant requirements. For this purpose, the structural engineer has to apply appropriate structural models, suitably-matched computational models and reliable structural parameters as close to reality as possible. Structural models and structural parameters have to be established on the basis of plans, drawings, measurements, observations, experiences, expert knowledge, codes and standards. Generally, certain information regarding structural models and precise values of structural parameters do not exist. Computational models must be capable of numerically simulating the system behavior of the chosen structural model. Mathematically exact solutions, however, are only available in exceptional cases. In general, weak solutions and approximations are used, internal parameters, e.g. in material laws, have to be defined and numerical solution techniques including lower bounds for numerical accuracy are applied. These facts show that structural engineering is significantly characterized by uncertainty. In order to perform realistic structural analysis and safety assessment this uncertainty must be appropriately taken into consideration.

Different methods are available for mathematically describing and quantifying uncertainty. Some of these basic concepts are e.g. probability theory [16], including subjective probability approach [31] and BAYES methods [29], interval mathematics [1], convex modeling [4], theory of rough sets [22], fuzzy set theory [2], theory of fuzzy random variables [15] and chaos theory [13]. In the scientific literature the new uncertainty models are not only controversially discussed [8] but also increasingly implemented for the solution of practice-relevant problems [27, 7, 3, 6, 12, 21, 30, 26]. These different developments of uncertainty models do not directly contradict each other but rather constitute an entirety.

The choice of an appropriate uncertainty model for solving a particular problem depends on the characteristic of the uncertainty present in the problem description and the boundary conditions. Most often, the well developed probabilistic models are applied to take account of uncertainty. For this purpose, random variables or random processes are generated for describing non-deterministic parameters or parameter fields. This presupposes assured and satisfactory statistical information to estimate the necessary probability distribution functions or special and sophisticated expert knowledge to assume prior distributions for a BAYESian approach. If these prerequisites for dependably applying probabilistic methods are not satisfied, it is advisable to make use of alternative uncertainty models. In this frequent case the engineer has to quantify structural parameters on the basis of only few data, which may additionally be characterized by vagueness, e.g. due to uncertain measurements or changing reproduction conditions. Moreover, some expert knowledge and linguistic assessments are required to be incorporated into the modeling. Hence, the engineer does only have an idea concerning the value range of these parameters and a kind of believe with which some values are more possible to occur than other ones. For modeling such information adequately a non-probabilistic uncertainty model that considers sets of parameter values together with subjective weighting information inside the set is needed. Fuzzy set theory provides the most powerful basis for this purpose. It permits set theoretical modeling of uncertain parameters and a subjective assessment of degrees with which the particular elements belong to the set by means of a membership function. This offers the chance for appropriately taking account of non-stochastic uncertainty, which frequently appears in engineering problems without making any artificial assumptions the validity of which cannot be proven.

This paper mainly focuses on the uncertainty model fuzziness. In order to provide some context standard Monte Carlo simulations and Monte Carlo simulations using response surfaces are also considered. The mathematical background of the uncertainty model fuzziness is addressed separately in this book of proceedings [19]. However, the background of the uncertainty model randomness and the probabilistic analysis methods are not considered herein since they are well known.

2 Deterministic computational model

The scope of the uncertainty investigations is an assembly of a vehicle body for a commercial van as shown in Fig. 1. The main components of that assembly are the first cross member and the front part of the longitudinal member of the frame. Furthermore, the absorbing box between the first cross member and the longitudinal member, parts of the wheelhouse and the closing panels of the longitudinal member are also included in the assembly.

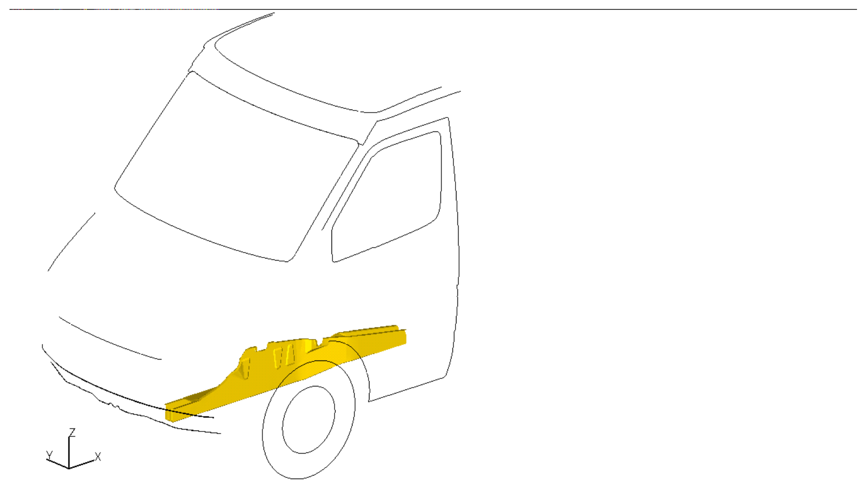


Figure 1: Position of the assembly within the vehicle

In Fig. 2, boundary conditions and part numbers of important components are illustrated. The assembly is connected to the bottom sheet of the vehicle body at the flanges of the front and of the back closing panels. In the simulations, the assembly is fixed in y- and z-directions at these flanges. The assembly is displaced with a constant velocity against a stonewall. The resulting deformations are numerically simulated

For simulation purposes, only a half of the frame is modeled (since geometry and loads are symmetric with respect to the y-z-plane), and the first cross member is cut off at the y-z-plane. Therefore, the translation in y-direction and the rotation about the x- and the z-axis have to be fixed along the cutting edge.

Evaluated responses are the internal energy and the stonewall force. The stonewall force is filtered using a SAE 180 filter. All results and parameters have been normalized.

Fig. 3 shows the deformed model.

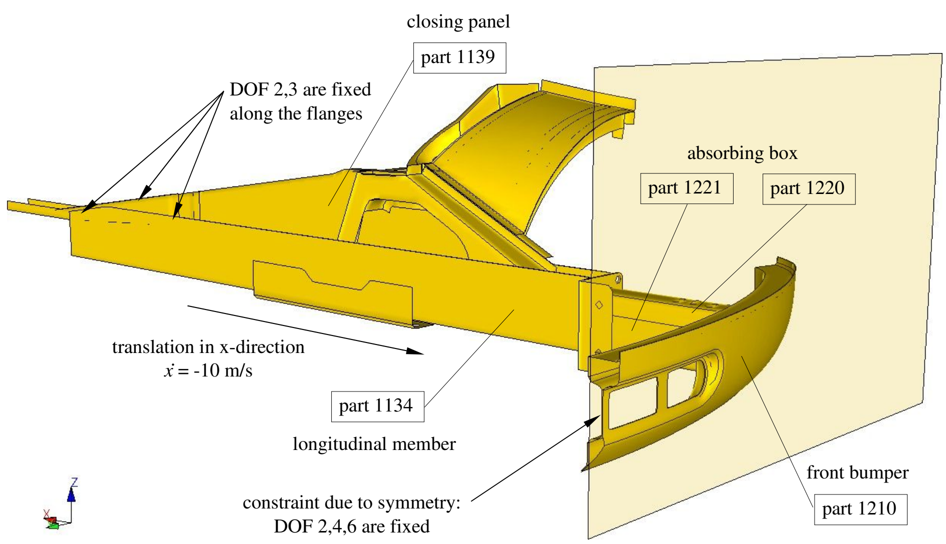


Figure 2: Boundary conditions and part numbers, the assembly is fixed in y- and z-directions at the flanges and displaced at a constant rate in x-direction

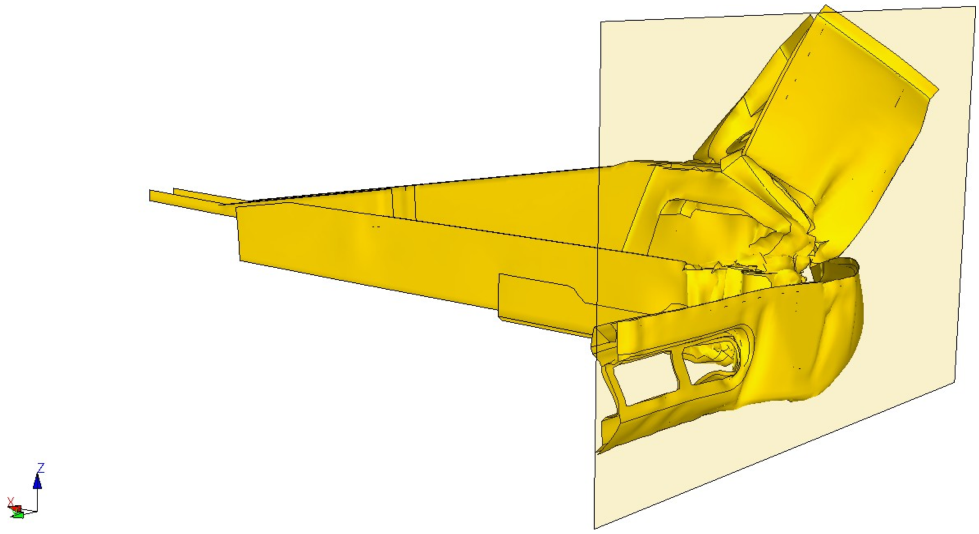


Figure 3: Simulation model - deformed configuration

3 Uncertainty quantification

3.1 Randomness

For the purpose of a stochastic analysis the sheet thickness of the closing panel (Part 1139), of the longitudinal member (Part 1134), of the absorbing box (1221) and of the front Bumper (Part 1210) as well as a scaling factor for the yield surface of the material of the longitudinal member (Part 1134) have all been modeled with normal distributions. The parameters used to describe these normal distributions are empiric.

An overview of the chosen sheet thicknesses and the parameters for the normal distributions are listed in Tab. 1. The mean values have been obtained by means of an optimization published in [25]. Based on an ANOVA [28] analysis of the points acquired during the optimization process the 4 sheet thicknesses with the highest regression coefficients were chosen. Furthermore, the yield surface of the longitudinal member shall be considered which was not used for the optimization but might have a significant contribution to the results and is characterized by a fair amount of uncertainty.

Table 1: Parameters used for stochastic analysis

description	parameter	mean	standard deviation
sheet thickness of longitudinal member	T 1139	1.0	0.020
sheet thickness of closing panel	T 1134	1.0	0.021
sheet thickness of front bumper	T 1210	1.0	0.062
sheet thickness of absorbing box	T 1221	1.0	0.050
scaling factor for yield surface	SF 1134	1.0	0.050

3.2 Fuzziness

For the description of the uncertain structural parameters that have been introduced in Sec. 3.1 only insufficient information is present. In particular, observations, measurements or statistical informations to estimate the needed probability distribution function types and associated parameters are not available. In order to perform a realistic structural analysis the uncertainty must be appropriately taken into consideration. The mathematical description of these uncertain structural parameters has to be realized on the basis of expert knowledge, i.e. experience. For modeling such information adequately the non-probabilistic uncertainty model fuzziness [18] is applied as an alternative. Fuzzy set theory [32] offers the most powerful basis for this purpose.

mathematical background A fuzzy set can be described as set of elements x which have a gradual membership. This gradual membership is represented by a membership function $\mu(x)$. Specifically a fuzzy set on X is defined as

$$\tilde{A} = \{(x, \mu_A(x)) \mid x \in \mathbf{X}\}. \tag{1}$$

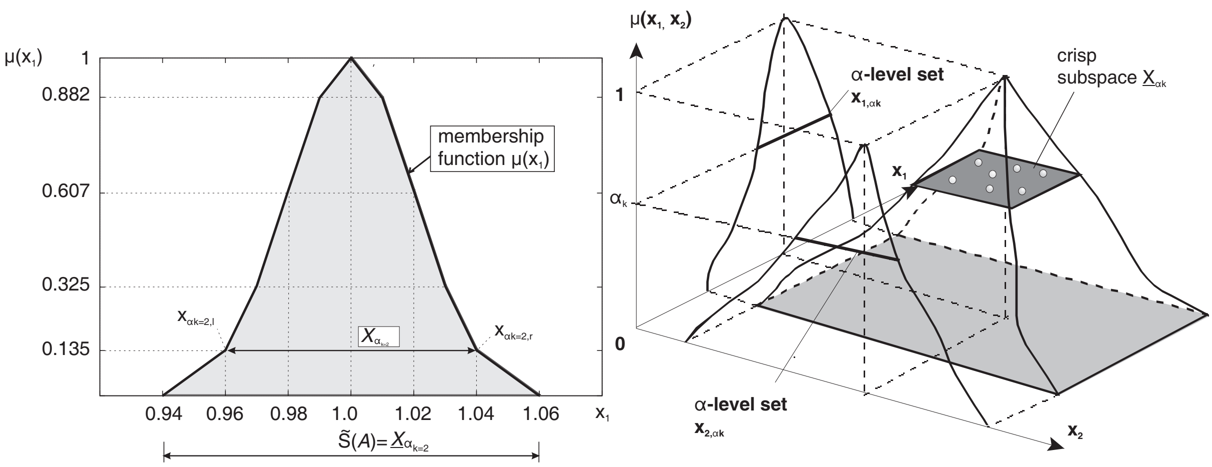
Fuzzy numbers are normalized fuzzy sets $\tilde{A}$ with a continuous and convex membership function. The membership function of a fuzzy number assesses precisely one element $x \in \tilde{A}$ with $\mu_A(x) = 1$.

To treat fuzzy numbers (subsequently also called fuzzy values) in a uncertain numerical analysis a discretization is necessary. The concept of α-discretization provides a numerically efficient representation of fuzzy sets. An α-level set A_{α_k} of the fuzzy set $\tilde{A}$ is defined as a crisp set

$$A_{\alpha_k} = \{x \in X \mid \mu_A(x) \geq \alpha_k\} \tag{2}$$

associated with a selected real number $\alpha_k \in (0,1]$. All α-level sets are crisp subsets of the support $S(\tilde{A})$. For several α-level sets of the same fuzzy set the following inclusion property holds

$$A_{\alpha_k} \subseteq A_{\alpha_1} \forall \quad \alpha_i; \alpha_k (0,\ 1], \quad \alpha_i \leq \alpha_k. \tag{3}$$



(a) Fuzzy set $\tilde{x}_1$

(b) Constitution of crisp subspaces $\underline{X}_\alpha$

Figure 4: α-level sets

For each α-level, the associated α-level sets A_{i,α_k} of the fuzzy input variables $\tilde{x}_i = \tilde{A}_i$ constitute an n-dimensional crisp subspace $\underline{X}_{\alpha_k}$ of the x-space, see Fig. 4 for a two-dimensional example. For the definition of the membership function of the fuzzy value $\tilde{x}_i$ two elements of the α-level set X_{α_k} are required. This elements represent the minimum $x_{i,\alpha_{k,l}}$ and the maximum $x_{i,\alpha_{k,r}}$ of the crisp α-level set X_{α_k} as marked in Fig. 4.

fuzzy structural parameters The uncertain structural parameters of the presented crash analysis example are modeled exclusively on basis of expert knowledge. The support of all fuzzy values $x_1 \ldots x_5$ compromises a possible value range. The shape of the membership function depends on an individual subjective assessment. The discretization is realized with six α-levels (see Tab. 2). The fuzzy value $\tilde{x}_1$ of the structural parameter T 1139 (for the fuzzy structural analysis) is exemplified in Fig. 4.

The application of the fuzzy structural analysis is described in Sec. 5.

Table 2: Fuzzy structural parameters

	T 1139		T 1134		T 1210	
$\alpha-$level	$x_{1,\alpha_k,l}$	$x_{1,\alpha_k,r}$	$x_{2,\alpha_k,l}$	$x_{2,\alpha_k,r}$	$x_{3,\alpha_k,l}$	$x_{3,\alpha_k,r}$
0	0.94	1.06	0.95375	1.0625	0.81	1.19
0.135	0.96	1.04	0.9583	1.0416	0.88	1.13
0.325	0.97	1.03	0.9687	1.0312	0.91	1.09
0.607	0.98	1.02	0.9791	1.0208	0.94	1.06
0.882	0.99	1.01	0.9895	1.0104	0.97	1.03
1	1.0	1.0	1.0	1	1	1

	T 1221		SF 1134	
$\alpha-$level	$x_{4,\alpha_k,l}$	$x_{4,\alpha_k,r}$	$x_{5,\alpha_k,l}$	$x_{5,\alpha_k,r}$
0	0.85	1.15	0.85	1.15
0.135	0.9	1.10	0.9	1.10
0.325	0.925	1.075	0.925	1.075
0.607	0.95	1.05	0.95	1.05
0.882	0.975	1.025	0.975	1.025
1	1	1	1	1

4 Stochastic structural analysis using LS-OPT

4.1 Monte-Carlo-Simulation

The Monte Carlo simulation is a widely used and accepted method to perform stochastic analysis. The main advantage of Monte Carlo simulations is the robustness and the reliability of its results. The main drawback is the huge amount of sampling points needed to obtain reasonably trustworthy results.

Nevertheless, the Monte Carlo simulation keeps being one of the most important tools in optimization and stochastic investigations and is very well suited to validate more advanced methods.

In this part of the investigation a Latin Hypercube sampling [28] has been used to generate the sampling points for a Monte Carlo simulation. Latin Hypercube sampling leads to more evenly distributed random numbers and avoids clustering by dividing the parameters into segments of equal probability. Thus, providing insight to the extremes of the probability distributions and reducing the amount of sampling points needed to obtain trustworthy results. A total number of 182 sampling points have been evaluated in this investigation.

Using LS-OPT a variety of different results can be evaluated which shall be introduced in the following to provide an overview of possible benefits of such stochastic investigations.

distributions and histograms Fig. 5 and Fig. 6 show histograms of the internal energy as well as the stonewall force. In addition, standard statistical values such as the mean and the standard deviation are calculated. Histograms are a good measure to get a first idea of the variance of the responses and can aid the engineer in the evaluation of the robustness of the system.

probability of failure Furthermore the probability of exceeding some given constraint can be computed. In Fig. 6 this is illustrated for the constraint:

$$F_{stonewall} < 1 \tag{4}$$

In addition to the probability of exceeding the constraint, a confidence interval is given to evaluate its trustworthiness. This confidence interval depends on the number n of sampling points used for the

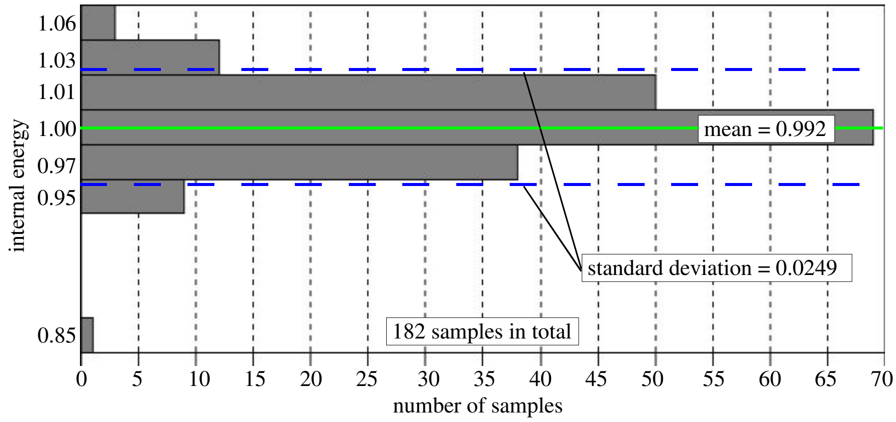


Figure 5: Histogram of internal energy and statistical parameters for a normal distribution

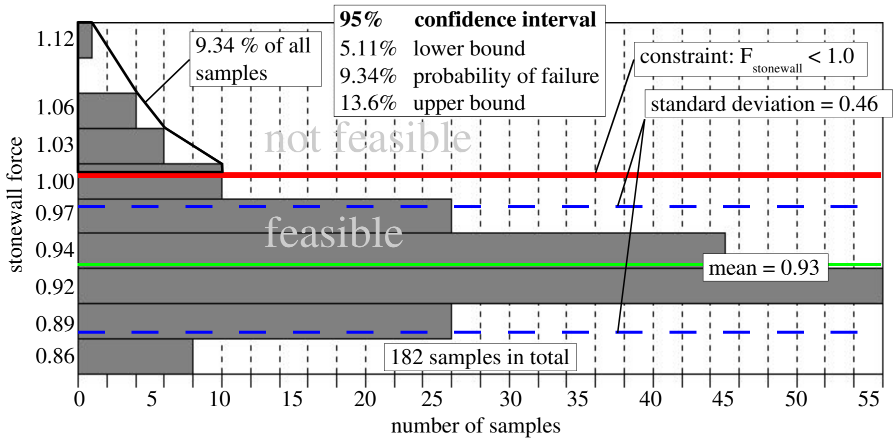


Figure 6: Histogram of stonewall force and statistical parameters for a normal distribution as well as a constraint upon the stonewall force

Monte Carlo simulation and on the desired probability that the true value lies within the confidence interval. Information on how to compute the confidence interval can be found in [17]. In Fig. 6 this is illustrated for a confidence interval of 95 %.

identification of bifurcation and clustering An easy way to identify model bifurcation is the evaluation of hill plots. In Fig. 7 the total internal energy is plotted over the sheet thickness of the longitudinal member. In this case we can clearly identify one outlier with considerably less energy absorption. A more detailed examining of the corresponding runs reveals that the differences in the internal energy are the result of different buckling modes.

Because of the multidimensionality of many problems the identification of bifurcations is not always as easy as in this example. For the outlier analysis of high dimensional problems one might consider using response surfaces and the determination of the distances of selected sampling points to this averaged solution of the problem (see Sec. 4.2). More sophisticated methods to separate bifurcation modes make use of cluster analysis algorithms [5, 11, 14].

statistics of histories In many cases it is desired to observe the evolution of a selected response and its statistical behavior in time. This aids the engineer to determine the cause of bifurcation and to understand the model behavior. Fig. 8 shows the stonewall force over time. One can see that the range between the mean and minimum curves increases significantly beginning with 60 ms. This is also the time at witch the buckling of the longitudinal member starts in run 47 causing the low stonewall

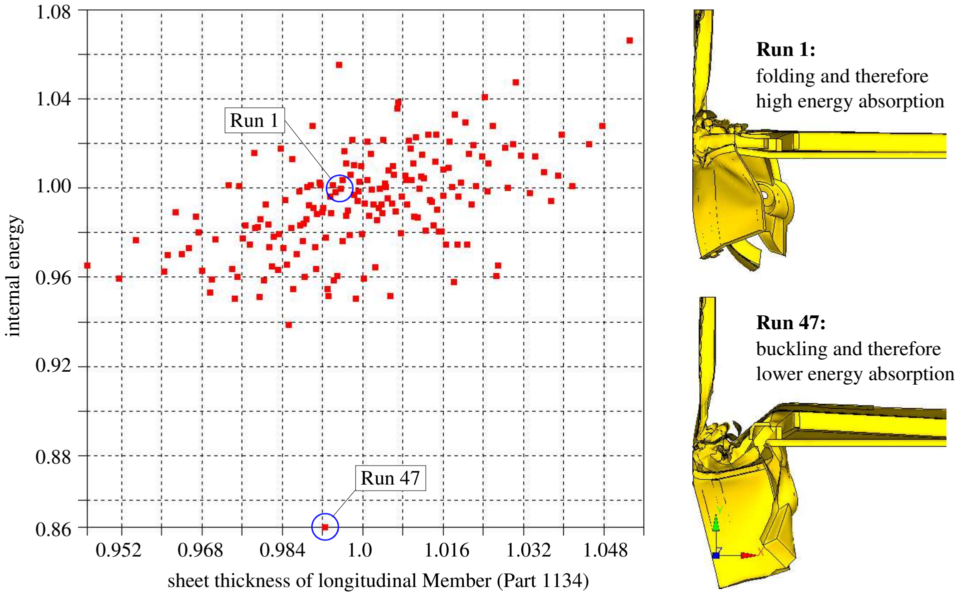


Figure 7: Ant Hill plot of internal energy over sheet thickness of longitudinal member, a bifurcation is clearly visible through the outlier run 47 where the longitudinal member is buckling globally instead of folding which leads to lower internal energies

forces. Using LS-OPT it is also possible to evaluate other statistical values such as standard deviations or quantiles.

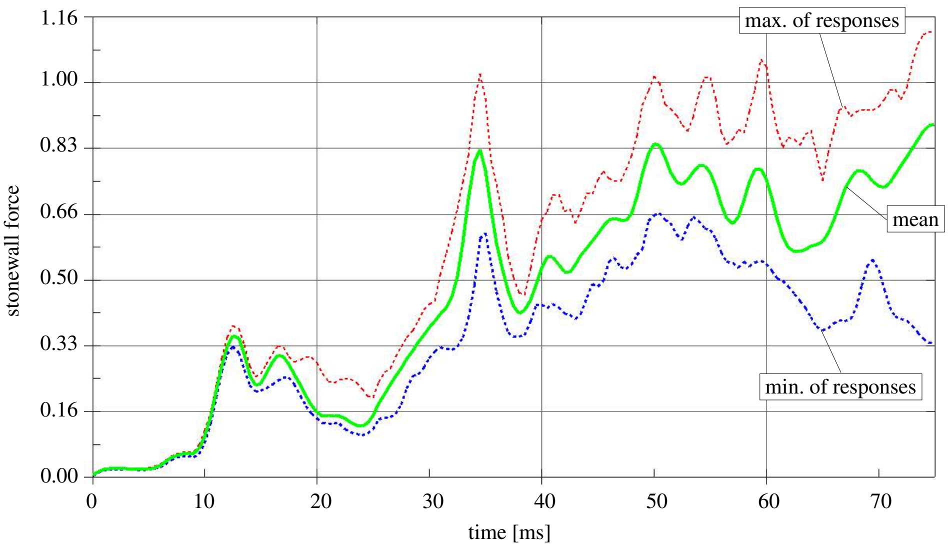


Figure 8: Time history of stonewall force and corresponding min. and max. values of all runs

visualization is fringe plots Additional insight into the model behavior can be achieved by fringe plots of statistical values over the FE net. Especially in order to identify the locations of bifurcation domains this can be a helpful tool. Fig. 9 shows such fringe plots of the displacement range of the y-component. In Fig. 9 only the undeformed absorbing box and the longitudinal member are shown, all other parts have been blanked. These fringe plots are not just static but can be observed for each time step using LS-PREPOST. This allows to determine not only the location of bifurcation but also the time of its occurrence.

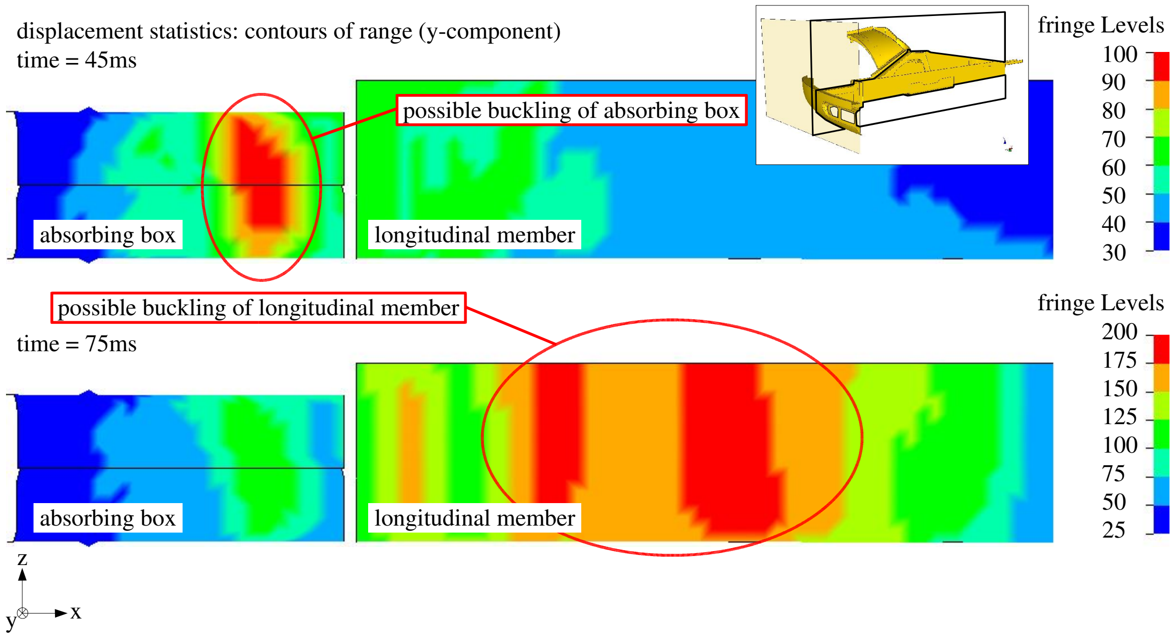


Figure 9: Statistical fringe plot of the displacement range of the y-component, this time dependent fringe plot allow the identification of the locations of bifurcation domains, alternatively other statistical parameters such as correlation coefficients could be fringed

identification of sensitivities (ANOVA) A means of determining the sensitivities of responses with respect to different parameters is an **An**alysis **o**f **Va**riance. This involves basically to create a linear response surface using a least square fit and then to evaluate the regression coefficients for different parameters as well as variance relative to this linear fit. In Fig. 10 we can see an ANOVA plot of LS-OPT for the internal energies.

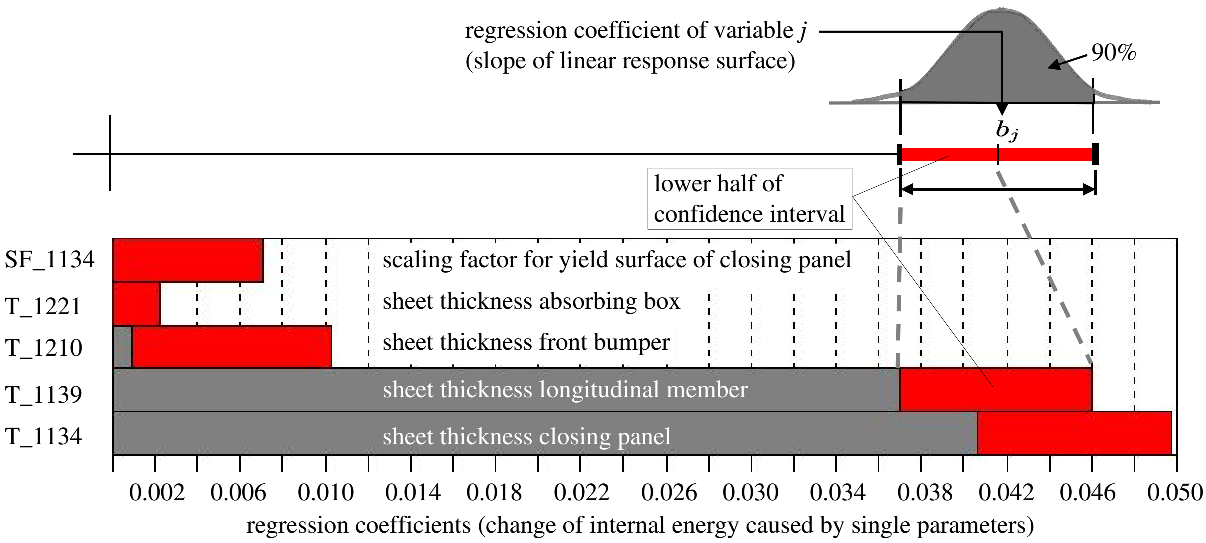


Figure 10: Analysis of Variance plot for the internal energy, only the sheet thicknesses of the closing panel, the longitudinal member and the front bumper can be considered significant

4.2 Monte-Carlo-Simulation using response surfaces

A meta model [28, 24, 20] is a way of representing a complex and computationally expensive problem by a simple and computational inexpensive surrogate model. The response surface of such a meta model can be used to perform the otherwise computationally very expensive Monte Carlo simulations.

LS-OPT offers a variety of different types of response surfaces. In order to use response surfaces one has to choose the type carefully. Linear response surfaces are not able to describe problems with a highly non-linear relationship between the model parameters and the responses. On the other hand, highly non-linear response surfaces might under certain conditions lead to an "over fitted" estimation of the problem. Generally, it is advisable to use linear response surfaces only if the considered design space is assumed to be mostly linear.

visualization of response surfaces Although the human mind cannot picture a surface of more than 3 dimensions the demand of visualizing multidimensional response surfaces from the engineers point of view is high. One approach is to reduce the problem to 2 or 3 dimensions and thus showing only a slice out of the multidimensional problem. The other parameters are fixed so that the curve or surface that is visualized only represents one iso-line or iso-surface of the multidimensional surface. This approach becomes useful when the engineer is able to interactively change the fixed parameters thus exploring the design space and getting a feeling for the influence of different parameters on the problem. For exploring the design space the visualization software D-SPEX is used, Fig. 11.

predictive capability of response surfaces In this example the stonewall force turned out to contain a huge amount of noise. In Fig. 11 this is illustrated. The shown neuronal net was trained using 100 sampling points in the 5-dimensional parameter space. In addition, there are 200 test points, which are not used to train the neuronal network. This allows to evaluate the predictive capabilities of the response surface. The test points vary only for the sheet thickness of the closing panel, the other parameters are fixed to their mean values (see Table 1). The same applies to the visualization of the response surface. Fig. 11 shows that the neuronal net is merely an estimation of the tendencies of the real response.

If too few sampling points are used the neuronal net cannot tell the difference between the actual physical response and the noise of the response. The response surface becomes overfitted. This improves if the number of points for the neuronal net is increased or the ratio of physical variance to noise is high within the chosen range. However, if the number of sampling points is limited the use of linear response surfaces is recommended especially for such narrow parameter ranges as in the presented example.

From the engineering point of view it would be advisable to consider a lower filter frequency for the stonewall force this would reduce the noise in the stonewall force and the deterministic relationship between the parameters and the response would dominate.

stochastic investigations using the response surface From Fig. 11 it becomes clear that using only the response surface in stochastic investigations leads to underestimated results since the sometimes huge amount of noise cannot be considered.

One way to consider the residuals is by evaluating them statistically and adding their variance to the variance of the responses obtained from the response surface as illustrated in Fig. 12. Further insight into this topic and the methods used in LS-OPT is given in [28, 23].

comparability of results As can be seen in Tab. 3 using response surfaces one can achieve results comparable to those obtained by standard stochastic procedures. The statistical results of a Monte Carlo simulation using a linear response surface build from 40 points are almost within the range of the confidence interval of a small Monte Carlo simulation using 182 points. Only the probability of failure is slightly outside the confidence interval of the Monte Carlo simulation. Confidence intervals for the results obtained with the aid of the response surface method have not been determined.

From the experience of this example one could deduce that for robustness analysis, in which the chosen parameter range is quite narrow and the expected amount of noise is high, it is advisable to use linear

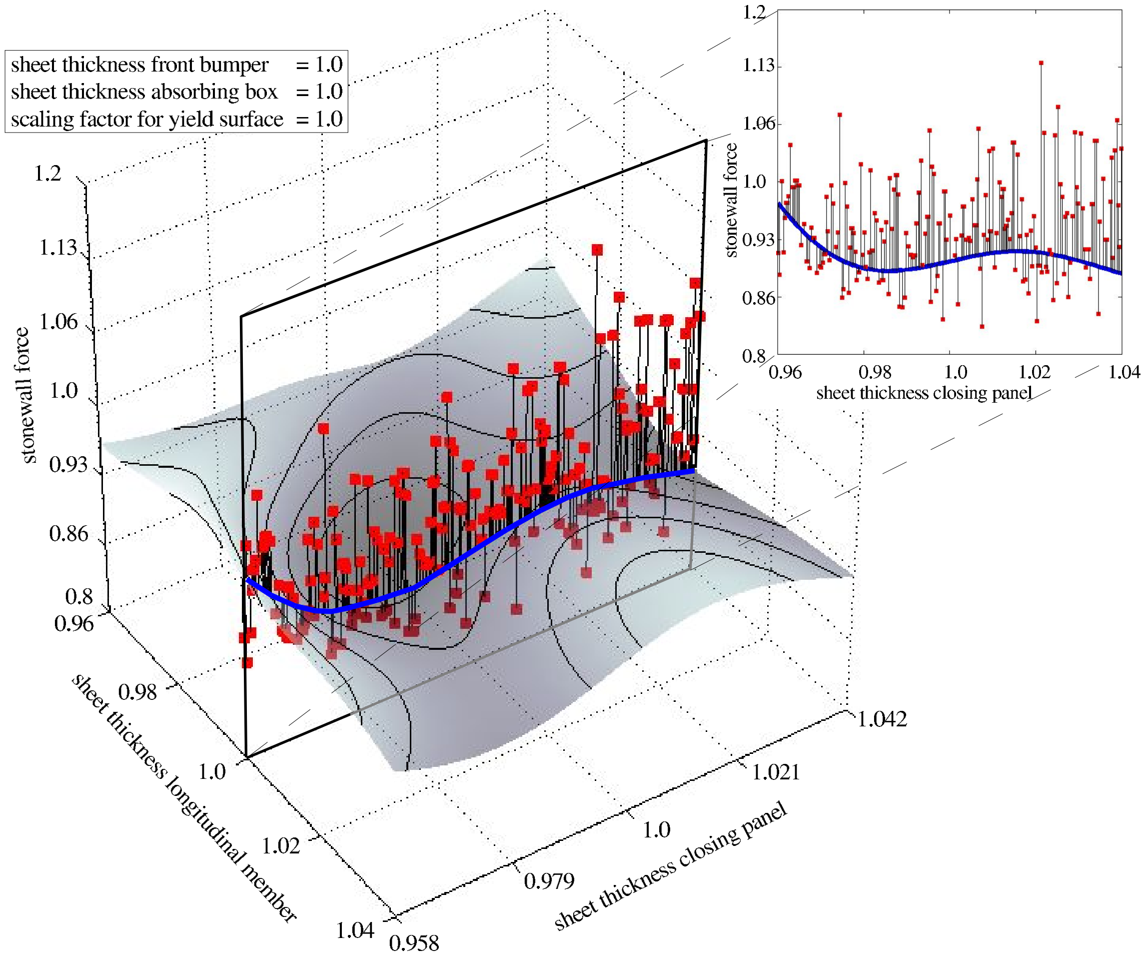


Figure 11: Neuronal network as a function of sheet thicknesses of longitudinal member and closing panel, the prediction error of the response surface is visualized for the 200 test points, that where not used to train the neuronal network

response surfaces for Monte Carlo simulations. The results, however, cannot resemble a Monte Carlo simulation especially since there is no such thing as a confidence interval. A further conclusion is that in order to perform statistical analysis on response surfaces there has to be a fair amount of oversampling. Only a sufficient oversampling allows to account for the noise of the response. This applies for the linear response surfaces as well as for higher order response surfaces.

5 Fuzzy structural analysis

In deterministic structural analysis crisp structural input parameters $\underline{x}$ representing loads, geometry, and material parameters are mapped with the aid of the computational model onto structural responses like, e. g. stresses, internal forces, or displacements. This mapping may denoted in the form

$$\underline{x} \rightarrow \underline{z} \tag{5}$$

A computational model, which characterizes the crisp dependency between the crisp vectors $\underline{x}$ and $\underline{z}$ is referred to as the *deterministic fundamental solution*. It represents the mapping model, indicated with the arrow.

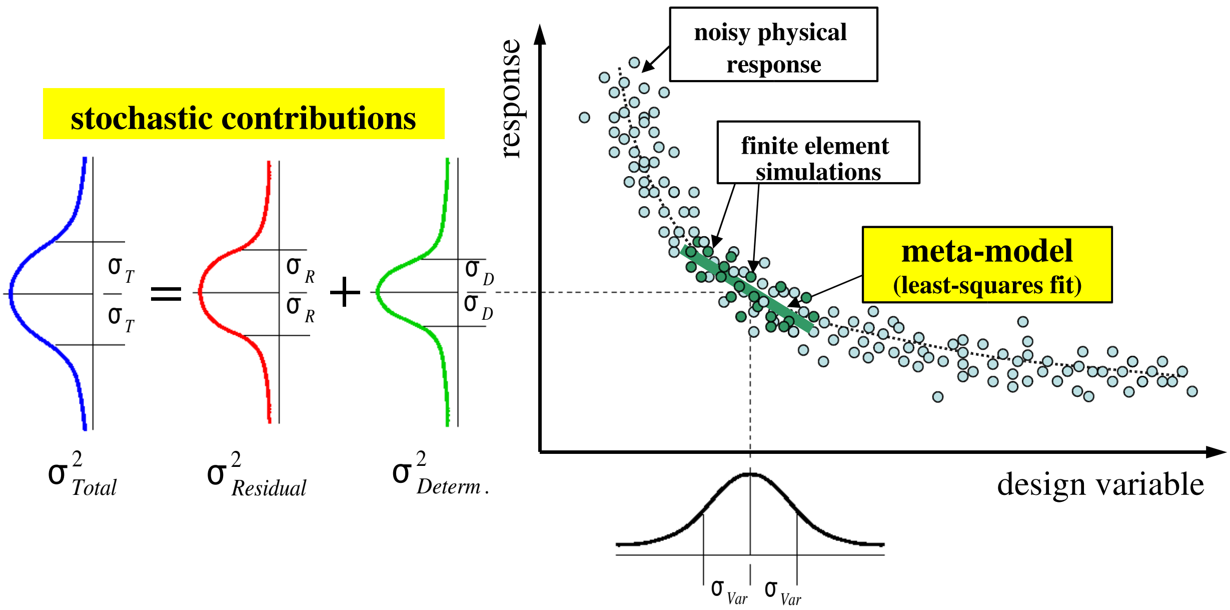


Figure 12: Consideration of residuals

Table 3: Monte Carlo vs. Monte Carlo on response surfaces (90% confidence interval)

	points	mean	dev.	confidence interval min.	max.	failure [%]	confidence interval min.	max.
internal energy								
Monte Carlo	182	0.99	0.024	0.022	0.027			
Monte Carlo	*1500*	*1.01*	*0.026*	*0.025*	*0.027*			
linear	20	0.99	0.037					
linear	40	1.00	0.024					
neuronal net	30	0.99	0.017					
neuronal net	100	1.01	0.023					
stonewall force								
Monte Carlo	182	0.94	0.047	0.042	0.051	9.3	5.1	13.6
Monte Carlo	*1500*	*0.94*	*0.047*	*0.045*	*0.048*	*10.8*	*8.77*	*12.7*
linear	20	0.97	0.049			3.6		
linear	40	0.98	0.050			14.2		
neuronal net	30	0.96	0.048			15.7		
neuronal net	100	0.94	0.053			12.9		

If structural parameters possess uncertainty, which can be identified as fuzziness the *fuzzy structural analysis* can be developed from Eq.5

$$\underset{\sim}{\tilde{x}} \rightarrow \underset{\sim}{\tilde{z}} \qquad (6)$$

Input vectors of the fuzzy structural analysis are then fuzzy structural parameters $\underset{\sim}{\tilde{x}}$. Fuzzy result vectors $\underset{\sim}{\tilde{z}}$ are determined on the basis of fundamental operations with fuzzy sets. The fuzziness of the uncertain structural parameters is processed on the basis of the developed α-level optimization [18, 19]. The solution technique is formulated in terms of a modified evolution strategy that targets at a minimal computational effort.

The concept of α-discretization provides a numerically efficient representation of fuzzy sets. For a sufficiently high number of α-levels a fuzzy set $\underset{\sim}{\tilde{A}}$ can be completely represented as a set of its α-level sets. All fuzzy input parameters are discretized using the same sufficient high number of number of α-levels $\alpha_k,\ k = 1 \ldots r$. With the aid of the deterministic fundamental solution (mapping model) it is possible to map crisp elements of the design space into the result space. The mapping of all elements of $\underline{X}_{\alpha_k}$ yields the crisp subspace Z_{α_k}.

Once the largest $z_{j,\alpha_{k_r}}$ and the smallest element $z_{j,\alpha_{k_l}}$ of the crisp subspace Z_{α_k} have been found, two points of the membership function of the fuzzy result z are known. The search for the smallest and largest result elements on each α-level represents an optimization problem and is referred to as α-level optimization.

The optimization problem is solved with a modified evolution strategy. The modified evolution strategy is a numerical evolution-based optimization method that is particularly suitable for solving α-level optimization within the scope of a general fuzzy analysis. It does not require any special properties of the objective function and is low-sensitive to noise. The numerical procedure possesses a simple structure and can be applied very flexibly in dependence on the problem by adjusting several effective control parameters. This concept permits an implementation of arbitrary non-linear algorithms as mapping models, e.g. for structural analysis, into a fuzzy analysis with α-level optimization.

The primary algorithm of the modified evolution strategy is formulated for continuous coordinates and constant constraints, which corresponds to a fuzzy analysis with non-interactive fuzzy input variables. An extension to more general conditions, in particular, for dealing with discrete optimization problems is straightforward.

The computational costs of the modified evolution strategy increases approximately linearly with the number of dimensions of the problem. The modified evolution strategy may be characterized as a generally applicable, numerically efficient and robust optimization technique. A post-computation is performed to improve the performance of the procedure. This combination represents a numerically efficient tool for fuzzy analysis. Fig. 13 shows the computation of α-level α_k by use of α-level optimization. The entire membership function of the fuzzy result z is determined α-level by α-level.

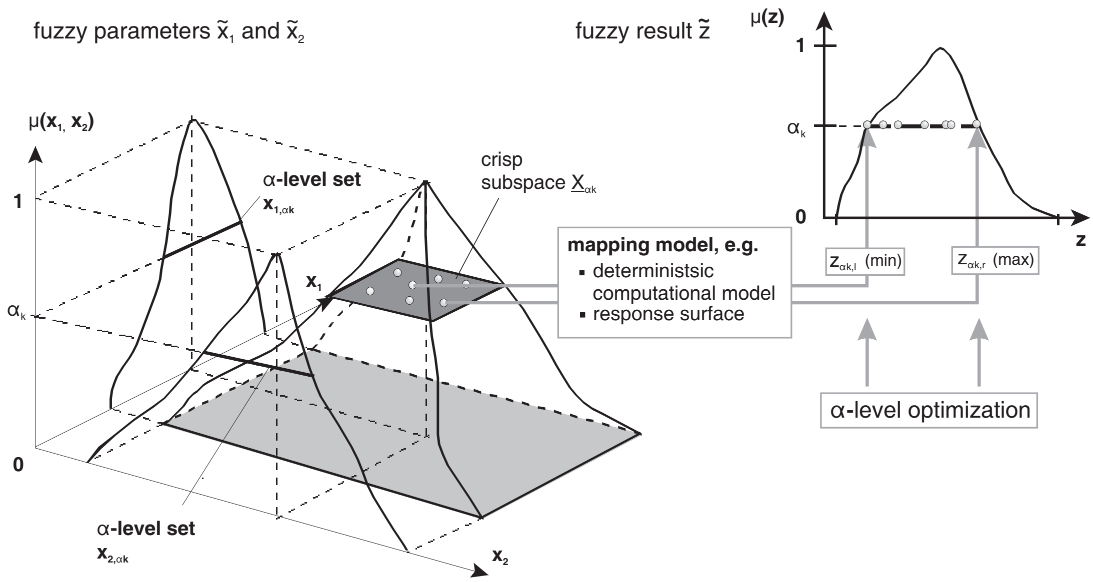


Figure 13: α-level optimization

Fuzzy structural analysis using a deterministic computational model The deterministic computational model as described in Sec. 2 is applied as deterministic fundamental solution for the fuzzy structural analysis. Eq. 5 may then also described by

$$\underline{\tilde{z}} = f(\underline{\tilde{x}}) \tag{7}$$

All fuzzified structural parameters quantified in Sec. 3.2 are input parameters of the fuzzy structural analysis. Fuzzy structural analysis then implies the analysis of the structure with the aid of the crisp algorithm – the so called determinisms fundamental solution – and consideration of fuzzy input parameters.

Fuzzy structural analysis using response surface In order to reduce the computational effort of the fuzzy structural analysis the deterministic computational model can be represented by a response surface, Fig. 11. In this study the neuronal network based response surface according Sec. 4.2 is adopted. Residuals are considered by adding their variance to the results obtained from the response surface. This meta model describes a random field, i. e., it possess the uncertainty randomness. The mapping of the fuzzy input parameters $\underline{\tilde{x}}$ onto the fuzzy result parameters $\underline{\tilde{z}}$ requires the selection of one realization of the random field. The fuzzy structural analysis can be formulated as

$$\underline{\tilde{z}} = f_{RS}(\underline{\tilde{x}}) \tag{8}$$

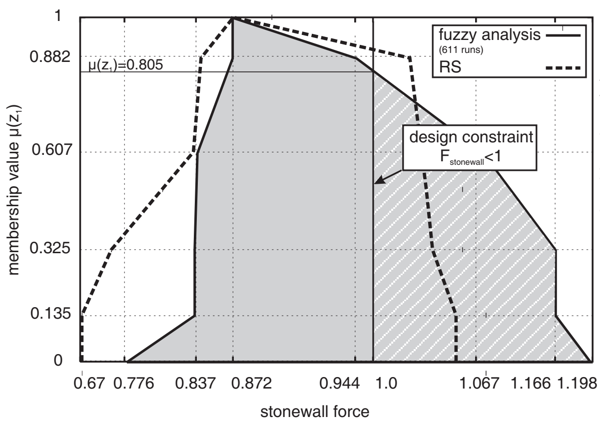
where f_{RS} denotes a realization of the random field.

5.1 Fuzzy structural analysis results

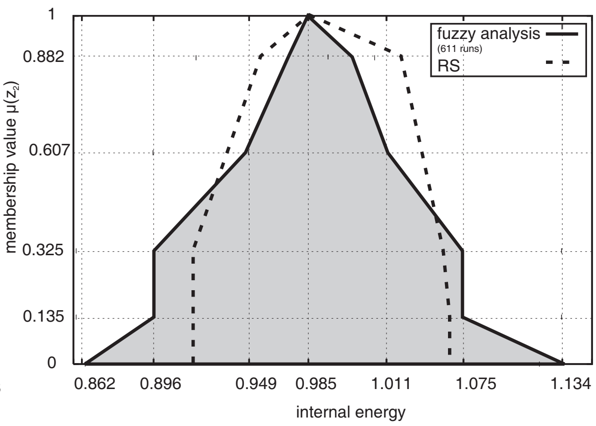
Fuzzy structural analysis using deterministic computational model In Fig. 14b the fuzzy analysis result of the *internal energy* using the deterministic computational model is shown (solid line). The membership function describes the degree of membership of all elements of the support $S(\tilde{z}_2) = (0.862; 1.134)$. Any analysis result that is obtained by mapping arbitrary points $\underline{x}_i$ from the space of fuzzy structural parameters $\underline{\tilde{x}} = \tilde{x}_1, \ldots, \tilde{x}_5$ onto the result space $\tilde{z}_i$ is at least member of the support.

The computed fuzzy stonewall force is shown in Fig. 14a. The design constraint according Eq. 4 subdivides the membership function (solid line) in permissible (0.776 until 1.0) and non-permissible values (> 1.0). All permissible values of the the fuzzy result (fuzzy stonewall force) hold dedicated permissible values of the fuzzy input parameters. The largest interrelated subspace of permissible fuzzy input parameters can be specified with the aid of the smallest α-level of the result membership function, that fulfill the design constraint. In this numerical study $\alpha = 0.805$ is obtained. If an exceeding of the design constraint has to be avoided strictly, the structural parameters have to be within the ranges given in Tab. 4.

The fuzzy complete processes of the internal energy and of the stonewall force are shown in Fig. 15. Decisive points of the process have been highlighted.



(a) Fuzzy stonewall force



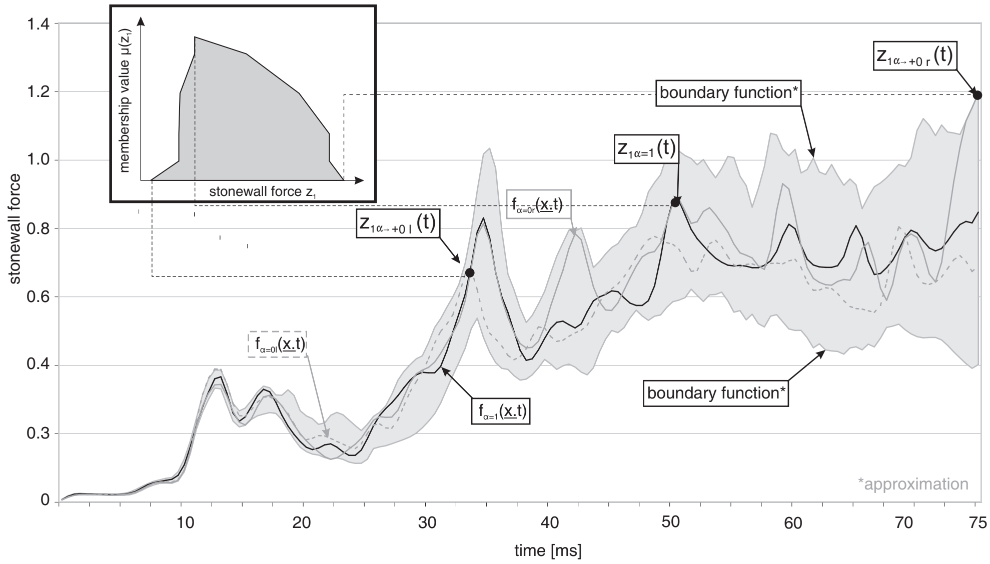
(b) Fuzzy internal energy

Figure 14: fuzzy analysis results

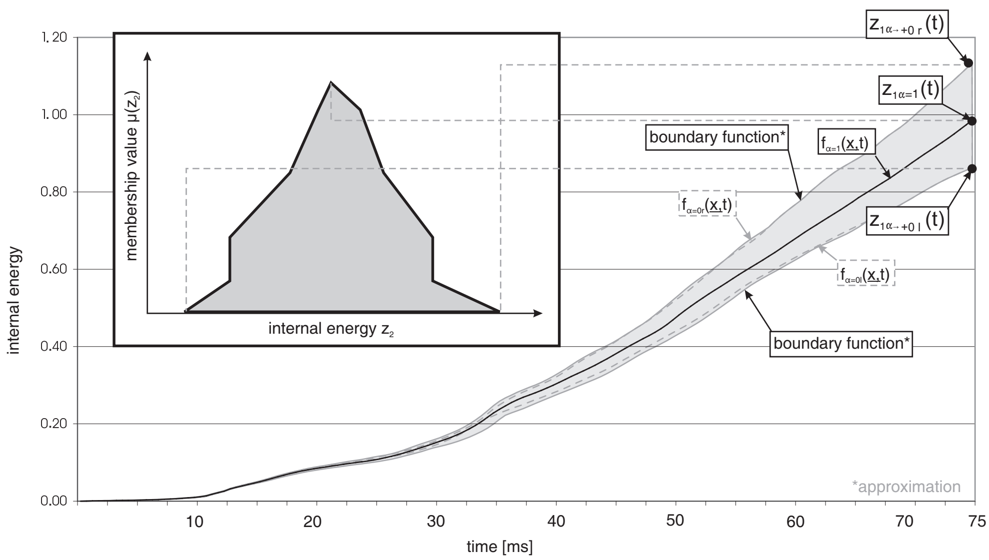
Fuzzy structural analysis using response surface The dashed lines in Fig. 14a and Fig. 14b are obtained by application of the fuzzy structural analysis using response surface. The difference between both membership function are caused by the selected realization of the random field. Each realization leads to an other membership function. The given fuzziness of the input parameters are strongly incorporated only by using the deterministic computational model.

Table 4: Permissible interval of structural parameters for design constraint $F_{stonewall} < 1$

	$x_{1,\alpha=0.805}$	$x_{2,\alpha=0.805}$	$x_{3,\alpha=0.805}$	$x_{4,\alpha=0.805}$	$x_{5,\alpha=0.805}$
lower boundary	0.99	0.9895	0.97	0.975	0.975
upper boundary	1.01	1.0104	1.03	1.025	1.025



(a) Fuzzy process of stonewall force



(b) Fuzzy process of internal energy

Figure 15: Fuzzy processes of the simulated structural responses z_1 and z_2

5.2 Result assessment

The support of a fuzzy number comprises a possible value range based on opinions of experts or expert groups, based on experience obtained from comparable problems and additional information. In some cases it might be possible to gain a partial influence of the uncertainty of structural design parameters support. The uncertainty of design parameters might be reduced by increasing quality control, by performing additional investigations; which is associated with additional financial effort. In such cases it is useful to know if reducing the uncertainty of the design parameters would have any substantial impact on the simulated results. In context of fuzziness and fuzzy analysis the membership is used to perform investigations that might be referred to as sensitivity or robustness analysis.

First of all a measure for uncertainty has to be introduced. The uncertainty of a fuzzy set $\tilde{A}_i$ can be assessed with an analog to SHANNONs entropy [9] which is defined by

$$\mathrm{H_u}(\tilde{A}_\mathrm{i}) = -\mathrm{k} \cdot \int_{\mathrm{x}=-\infty}^{\mathrm{x}=+\infty} [\mu(\mathrm{x}) \cdot \ln(\mu(\mathrm{x})) + (1-\mu(\mathrm{x})) \cdot \ln(1-\mu(\mathrm{x}))]\, \mathrm{dx} \tag{9}$$

The Shannon's entropy represents the "steepness" of the membership function. When assessing a crisp set the measure value $H_u(\tilde{A}_i) = 0$ is obtained. The most uncertain set with all its elements assessed by the membership value of $\mu(x) = 0.5$ (except the mean value) yields a maximum measure value $H_u(\tilde{A}_i)$.

A relative sensitivity measure is defined by the ratio of the modified Shannon's entropy of the fuzzy design parameters $\underline{\tilde{x}}$ to the SHANNONs entropy of the fuzzy result $\tilde{z}_j$

$$B_j^s = \sum_k^n \frac{H_u(\tilde{x}_k)}{H_u(\tilde{z}_j)} \tag{10}$$

The development of the normalized sensitivity measure $\overline{B}_1^s$ and $\overline{B}_2^s$ for the fuzzy results $\tilde{z}_1$ and $\tilde{z}_2$ is plotted in Fig. 16 over the respective α-value. An increasing α-level means a decreasing uncertainty of the structural design parameters. An ascending value for the sensitivity measure indicates a sensitive range. On α-level $\alpha = 0.325$ a rise of the sensitivity measure B_j^s can be identified. Not until restraining the design space of structural parameters to the ranges on α-level $\alpha = 0.325$ (Tab. 2) the structure responses are comparatively more sensitive.

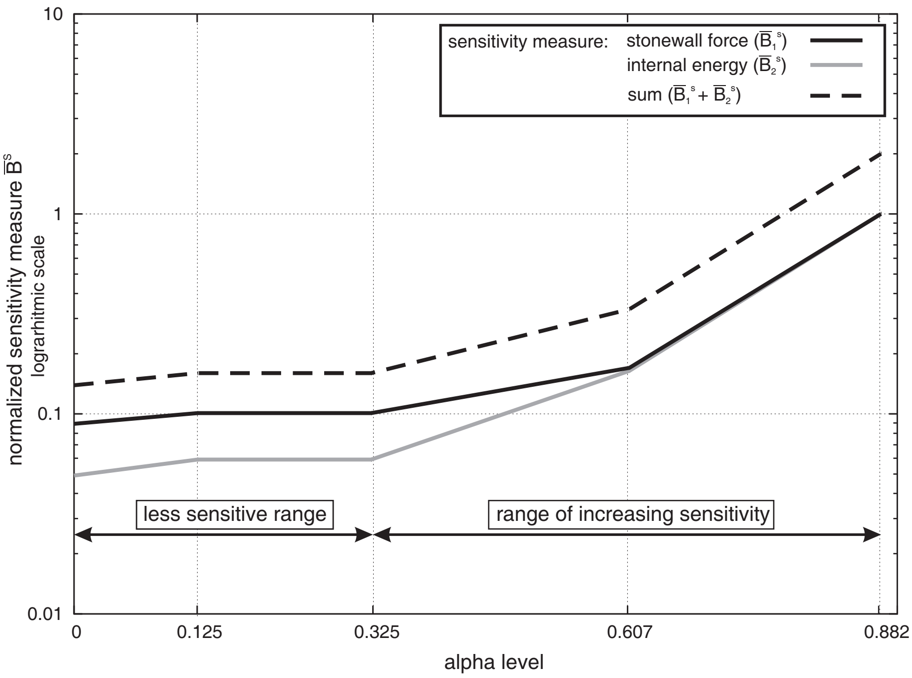


Figure 16: Sensitivity of the simulated structural responses

6 Conclusions

In this paper an uncertainty investigation of a crash analysis example is presented. The uncertainty models randomness and fuzziness are applied. The structural design parameters of the presented crash analysis example do not permit a mathematically assured description as random variables. Only insufficient information is present. In particular, observations, measurements or statistical informations are not available. In the case of randomness the probability distribution types and the associated parameters have to be assumed. The modeling of the fuzzy parameters is also based up on assumptions but the model fuzziness describes the uncertainty rather than the model randomness. The significance and the authenticity of the results are in both cases restricted to this assumptions.

The introduced uncertainty model fuzziness admits a new way to appropriately taking into account uncertainty that does not possess the characteristic of randomness and does not satisfy the prerequisites of the probabilistic concept. The algorithm of fuzzy structural analysis is a numerically efficient tool for uncertainty processing that is particularly suitable for non-linear engineering problems. The entire uncertainty of the structural design parameters is transfered to the results and allows a realistic and reliable assessment. Robustness or, as shown in this paper, sensitivity analysis is realized on the basis of SHANNONs entropy. Fuzzy analysis is also appropriate to perform worst case and best case studies with the presence of uncertainty.

The concept of fuzzy structural analysis as presented in this paper and in [18, 19] is advantageous compared with most of the fuzzy structural analysis based on fuzzy set theory. Most of the available methods rely on vertex methods. Due to the fact that only the interval bounds or corner points of the processed α-levels are taken into consideration during the analysis, these methods are restricted to monotonic problems. The transformation method presented in [10] is as a further development of the vertex method. It is approved to be right that this method is no longer restricted to monotonic problems but among other things its numerical effort increases exponentially with the number of fuzzy input variables and prevents this method to be applied for real world problems.

Table 5: Fuzzy analysis vs. Monte Carlo simulation

	Fuzzy analysis	Monte Carlo simulation
prerequisite for uncertainty quantification	. quantification on basis of expert knowledge, experience, measurements and technological inputs; non-stochastic uncertainty can be quantified	. statistical assured information (measurements etc.) are necessary
computational effort	. increases with the number of considered number of analysis results and input parameters . results of different degree of uncertainty are obtained with a one fuzzy analysis	. is independent from considered number of analysis results and input parameters . varying distribution parameters demand additional runs
results	. assessed value ranges of all possible results are obtained . best/worst case studies can be performed without additional effort . direct determination of permissible ranges of the structural input parameters	. probability distribution functions are obtained . determination of failure probabilities . value range of frequently appearing results

The computational effort of the fuzzy algorithms discussed in this paper is comparable to or better than standard Monte Carlo simulations but the results are restricted to one response at a time. Evaluation of further responses always will need additional sampling points. The gain in efficiency using response surfaces for Monte Carlo simulations can be substantial and the obtained results are well comparable.

The same gain in efficiency is true if the response surfaces are applied for the fuzzy algorithms. In this case the computational effort of both methods is exactly the same due to the inexpensive evaluations on the response surface. Evaluation of further responses will not require additional sampling points since a response surface can be build for every possible response from the available calculations.

Both methods seem to perform best if the amount of noise in the response is low. The problem of separating noise from physical behavior has been extensively addressed [23] for Monte Carlo simulations using response surfaces. Using response surfaces it seems substantial to consider the noise of the response represented by the residuals in order not to underestimate the uncertainty. Advantages and disadvantages have been condensed and listed in Tab. 5. But it has to kept in mind that both analysis methods are based on different uncertainty models that provide different information and obey unequal mathematical definitions and regulations.

Based on enhanced uncertainty concepts fuzzy variables and random variables may also be processed simultaneously. For this purpose the generalized uncertainty model *fuzzy randomness* [18] can be applied, which — additionally — enables the treatment of *fuzzy random variables*.

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