Structural health monitoring under consideration of uncertain data

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Abstract:

In the paper an approach for structural health monitoring is introduced comprising uncertain structural analysis, assessment of damage and performance by uncertainty-indicators as well as prognosis of structural behavior and lifetime. This approach is referred to as numerical structural monitoring and may be supported by results of in-situ monitoring. Numerical structural monitoring is based on nonlinear structural analysis considering the comprehensive load and modification process. This process generally depends on uncertain data which are assessed by means of imprecise probabilities. Here, the imprecision is modeled with fuzzy parameters. This leads to the generalized uncertainty model fuzzy randomness representing the basis for the uncertainty measure fuzzy probability. A fuzzy stochastic analysis considering time-variant uncertain data has been developed to take into account fuzzy randomness of the load and modification process within the numerical structural analysis.

Keywords:

numerical structural monitoring; reliability assessment; uncertain data; imprecise probability; structural indicators.

1 Introduction

Structural health monitoring means the lifetime-oriented analysis of structures. Extending the conventional in-situ monitoring which bases on experimental analysis the approach introduced here focus on numerical structural analysis. This approach is referred to as numerical structural monitoring and enables a profound estimation of future structural behavior beside the computational reproduction of the occurred load and modification process. As a result, prognoses of the time-dependent structural behavior and safety are provided. Thus, reasonable statements on necessity, kind, and benefit of reconstructions can be made.

Numerical structural monitoring may be accompanied by in-situ-monitoring. Thereby, structural responses such as displacement of certain structural points are measured continuously or in intervals, [1]. However, in-situ monitoring is expensive and complex in comparison to a numerical structural analysis, [5]. Determining the results of interest by numerical structural analysis can reduce the total expenditure of structural health monitoring. Furthermore, adulterating influences can be detected comparing results of both, numerical and in-situ monitoring.

Considering bridges as an example, the measurement of displacement requires the blocking of the transport link and expensive static and dynamic load tests. Without obstructions of traffic, the damage state can only be detected integrally by analysis of the eigenfrequency and/or stiffness. However, this method is unable to capture each local effect in detail which may cause an exceeding of limit states, [10]. A combination of in-situ and numerical monitoring could cover these failure points as well.

To analyze the structure realistically, the complete structural process needs to be considered. This includes, e.g., time-dependent changes in material, geometry, and loading. The consideration of the load and modification process requires an applicable, in general numerical static and dynamic analysis which captures geometrical and physical nonlinearities, enables load-path-dependent material models, includes long-term effects as aging and damage, and admits the modeling of arbitrary system modifications.

In general, parameters of structural processes are uncertain. The uncertainty results from various reasons, e.g., external loads caused by different independent events are modeled with only a few load parameters without reflecting the exact time and space dependency. Strengths of materials are assessed integrally disregarding the space-dependent microstructure. Geometrical measured data are subjected to inaccuracies. Frequently, variables are described utilizing subjective assumptions. In consequence, it is important to consider uncertainty in numerical structural monitoring. The description of uncertain variables is traditionally realized by stochastic data models. However, parameters of structural processes often possess properties which require generalized uncertainty models. Especially, if only few sample elements are available for a parameter, and the samples are not gained under constant conditions a pure stochastic approach is inappropriate.

In this paper the generalized uncertainty model fuzzy randomness, which includes fuzziness and randomness as special cases, is presented. After some remarks on structural processes in section 2.1 the uncertainty models randomness, fuzziness, and fuzzy randomness are summarized in section 2.2. The mathematical basis of fuzzy and fuzzy random functions to describe the discontinuous structural process follows in section 2.3. The procedure of numerical monitoring is outlined in section 3. Definitions of structural indicators in section 3.1 enable an assessment of structural processes. The numerical realization is explained in section 3.2. The application of the presented approach is demonstrated by a T-beam bridge in section 4.

2 Uncertain processes within the numerical structural monitoring

2.1 Structural process and structural modification

A structure is subject to numerous alterations during its lifetime. These structural alterations are referred to as “structural modification”. The structural modification may result from sequence of different states during construction, changes in geometry and material, e.g., due to physical and chemical processes or changes in load, and accidental actions. It comprises cross section modification and modification of structural members and support conditions [3].

Because of its time-dependency, the entirety of loads and structural modifications constitutes the discontinuous structural process. Analyzing a structure during the lifetime close to reality requires considering the complete structural process. The analysis leads to time-dependent, discontinuous result processes

$$\underline{z}(\underline{t}) = f\left(g(\underline{t}), p(\underline{t}), \underline{A}(\underline{t}), \underline{I}(\underline{t}), \underline{E}(\underline{t})\right) \tag{1}$$

with

$\underline{z}(\underline{t})$	structural responses (e.g., displacements and internal forces)
$g(\underline{t})$	dead load
$p(\underline{t})$	static and dynamic external loads
$\underline{A}(\underline{t})$, $\underline{I}(\underline{t})$	parameters of geometry (e.g., cross sections, dimensions of the system, location of the reinforcement, and the prestressing elements)
$\underline{E}(\underline{t})$	material parameters
$\underline{t} = (\underline{\theta}, \tau, \underline{\varphi})$	spatial coordinates $\underline{\theta} = (\theta_1, \theta_2, \theta_3)$, time τ, further parameters $\underline{\varphi}$, e.g., temperature.

2.2 Remarks on uncertainty

The data uncertainty of the structural process have to be modeled according to there origin in order to obtain meaningful results [6]. The mathematical models

- Randomness
- Fuzziness
- Fuzzy randomness

are appropriate to describe data uncertainty (see Figure 1). Thereby, fuzziness and randomness are considered as special cases of the generalized model fuzzy randomness that encloses both stochastic and non-stochastic properties of parameters.

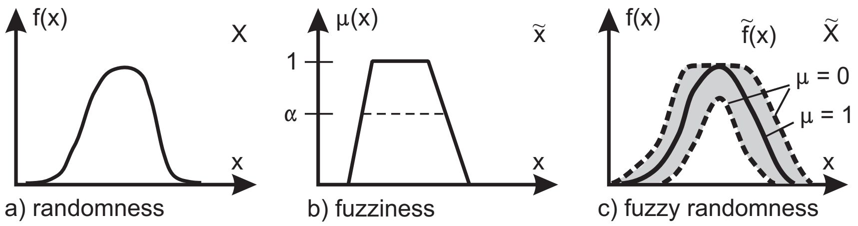


Figure 1: Mathematical models of uncertainty

The selection of the model depends on the available data. If sufficient statistical data exist, a parameter is usually described stochastically. Thereby, the choice of the type of the probability distribution function affects the result considerably. However, the type of the probability distribution function cannot be determined definitely also in the case of large samples. Thus, the assumptions of the type bases partly on subjective assessments. Moreover, the data for parameters are frequently fragmentary and imprecise. Then, describing the uncertainty with the model fuzzy randomness is recommended. Fuzzy randomness is also suitable if, e.g., reproduction conditions vary during the period of observation, or if expert knowledge complements the statistical evaluable material.

2.3 Uncertain processes in lifetime

Uncertain parameters of the structural process may be quantified as random, fuzzy, and fuzzy random variables according to the selected uncertainty model. In the case of time-dependency the parameters are described mathematically by random, fuzzy, and fuzzy random processes. Some basics to fuzzy and fuzzy random processes are outlined in the following. Thereby, fuzzy random processes contain the established random processes as a special case.

Fuzzy Processes. A time-dependent fuzzy structural parameter $\tilde{x}(\tau)$ may be understood as fuzzy process $\tilde{x}(\tau)$, whose functional values are fuzzy values $\tilde{x}_\tau \in \mathbf{F}(\mathbf{X})$ for each argument τ, see [2, 6].

$$\tilde{x}(\tau) = \{\tilde{x}_\tau = \tilde{x}(\tau) \;\; \forall \quad \tau \mid \tilde{x}_\tau \in \mathbf{F}(\mathbf{X})\} \tag{2}$$

Fuzzy processes $\tilde{x}(\tau)$ may be described with the aid of a crisp functional relationship in dependency of fuzzy bunch parameters $\underline{\tilde{s}}$ and the time τ. Therewith, the fuzzy bunch parameter representation of fuzzy processes is obtained.

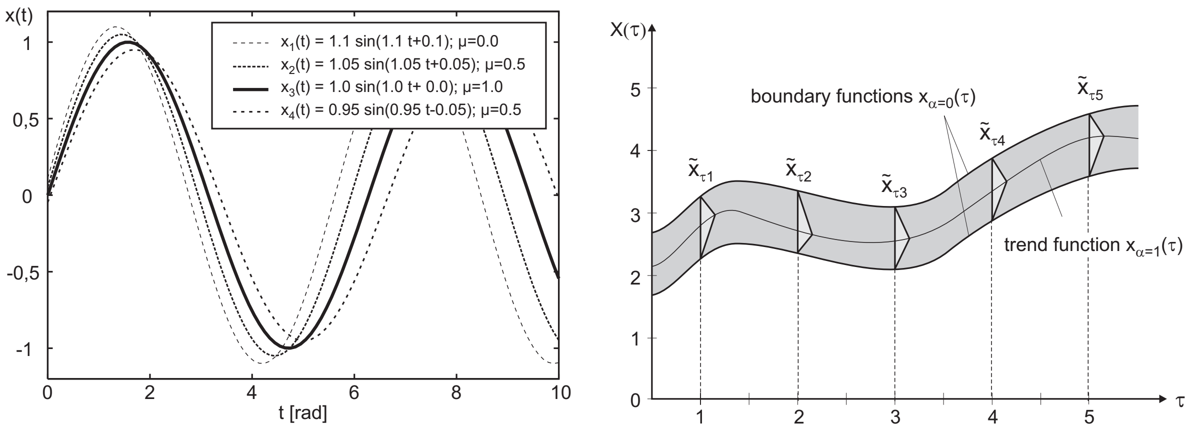
$$\tilde{x}(\tau) = x(\underline{\tilde{s}}, \tau) = \{\tilde{x}_\tau = x(\underline{\tilde{s}}, \tau) \ \ \forall \quad \tau \mid \tilde{x}_\tau \in \mathbf{F}(\mathbf{X})\} \tag{3}$$

Each real vector of bunch parameters $\underline{s} \in \underline{\tilde{s}}$ with membership values $\mu(\underline{s})$ (Figure 1b) determines a real-valued process $x(\tau) = x(\underline{s}, \tau) \in \tilde{x}(\tau)$ with membership $\mu(x(\tau)) = \mu(\underline{s})$. Thus, fuzzy processes may be interpreted as a fuzzy set of all real-valued processes $x(\tau) \in \tilde{x}(\tau)$. These real-valued processes $x(\tau) \in \tilde{x}(\tau)$ are referred to as trajectories of the fuzzy process.

A one-dimensional fuzzy process is demonstrated by

$$\tilde{x}(t) = x(\underline{\tilde{s}}, t) = \tilde{a} \cdot \sin\left(\tilde{\omega}_0 \cdot t + \tilde{b}\right) \quad \text{with} \quad \underline{\tilde{s}} = \{\tilde{a}, \tilde{\omega}_0, \tilde{b}\}. \tag{4}$$

This process depends on the fuzzy bunch parameters $\tilde{a} = \langle 0.9; 1.0; 1.1\rangle$ (fuzzy amplitude), $\tilde{\omega}_0 = \langle 0.9; 1.0; 1.1\rangle$ (fuzzy frequency), and $\tilde{b} = \langle -0.1; 0.0; 0.1\rangle$ (fuzzy phase shift). Selected real-valued processes are shown in Figure 2a for four specific combinations of a, ω_0 and b. In contrast to this continuous representation, a discretized fuzzy process with the fuzzy functional values $\tilde{x}_{\tau_1}, \ldots, \tilde{x}_{\tau_5}$ is shown in Figure 2b.



(a) Fuzzy process in fuzzy bunch parameter representation (b) Discretisized fuzzy process

Figure 2: Fuzzy processes

Both fuzzy bunch parameters $\underline{\tilde{s}}$ of the time-dependent fuzzy input parameters $x(\underline{\tilde{s}}, t)$ and the discrete fuzzy functional values $\{\tilde{x}_\tau \in \mathbf{F}(\mathbf{X})\}$ may be considered as input parameters.

Fuzzy Random Processes. Structural parameters which exhibit both fuzziness and randomness are modeled with fuzzy random variables $\tilde{X}$ or, in the case of time-dependency, with fuzzy random processes

$$\tilde{X}(\tau) = \left\{\tilde{X}_\tau = \tilde{X}(\tau) \ \ \forall \quad \tau\right\}, \tag{5}$$

see [6, 11]. The functional values of a fuzzy random process $\tilde{X}(\tau)$ are fuzzy random variables. At each time point τ the fuzzy random variables $\tilde{X}_\tau$ may be described by means of a fuzzy probability density function $f(x, \underline{\tilde{s}}, \tau)$ (see Figure 1c) and the respective fuzzy probability distribution function $F(x, \underline{\tilde{s}}, \tau)$. A fuzzy probability distribution function at a specific time point τ_1 is a fuzzy function. Figure 3 illustrates a GAUSSian distributed fuzzy probability distribution function $F(x, \underline{\tilde{s}}, \tau_1)$ of a time-dependent fuzzy structural parameter $\tilde{X}(\tau_1)$ at time τ_1 which depends on the fuzzy standard deviation $\tilde{\sigma}$ and fuzzy mean value $\tilde{m}_x$. In general, modeling of the distribution parameters as fuzzy values $\tilde{\sigma}$, $\tilde{m}_x$ is based on uncertain data analysis and subjective assessment. The fuzzy values are combined to the fuzzy bunch parameter vector $\underline{\tilde{s}} = (\tilde{\sigma}, \tilde{m}_x)$. Then, the fuzzy probability distribution function of a GAUSSian distribution reads

$$F_\tau(x, \tilde{s}) = \frac{1}{\tilde{\sigma} \cdot \sqrt{2\pi}} \cdot \int_{-\infty}^{x} e^{-\frac{1}{2} \cdot \left(\frac{x - \tilde{m}_x}{\tilde{\sigma}}\right)^2} dx \tag{6}$$

and represents the assessed set (fuzzy set) of all possible real-valued probability distribution functions. The fuzzy bunch parameter representation of fuzzy probability distribution functions might be applied independently of the distribution type, e.g., WEIBULL, GUMBEL, beta, and exponential distributions may be modeled as well.

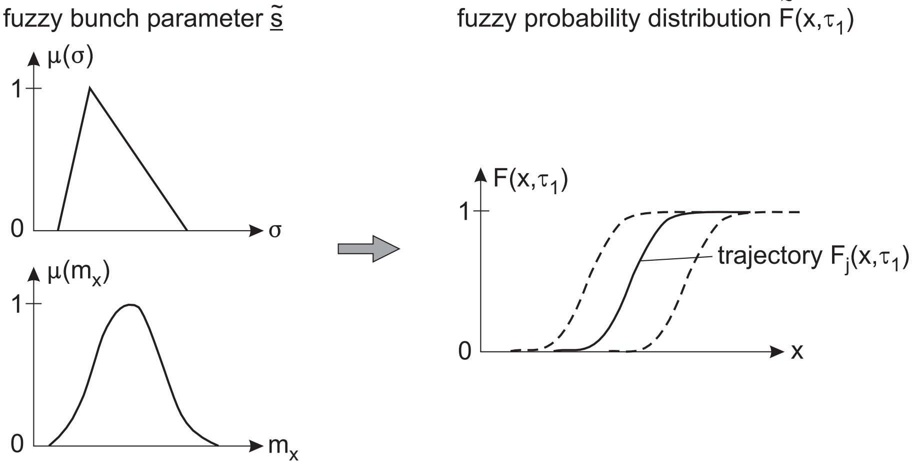


Figure 3: Fuzzy probability distribution function of the fuzzy random design parameter $\tilde{X}_{\tau_1}$

Numerical processing of a fuzzy random process $\tilde{X}(\tau) = X(\underline{\tilde{s}},\tau)$ requires the discretization of their time argument τ, see section 3.2.

3 Indicators and algorithms for numerical structural monitoring

Monitoring is the sequential gathering of information about the structural state and load exposure over the time. The intention of monitoring is to detect changes in bearing capacity, indicate deterioration, and plan necessary reconstructions. State-of-the-art is in-situ monitoring which allows the subsequent interpretation of the occurred structural processes and a respective assessment.

An alternative or rather addition to in-situ monitoring is the prognostic numerical structural monitoring. Thereby, a prognosis of the expected structural behavior is made on the basis of numerical structural analysis. Different paths of structural behavior could be simulated under assumption of potential scenarios, e.g., consequences of conversions, different prognoses on load trends, several kinds of modification processes, etc. This enables the qualitative and quantitative assessment of potential reconstruction measures and the respective appropriate point in time. Advantageously, numerical monitoring can always be performed, independently from real time.

3.1 Indicators for the assessment of uncertain structural processes

In principle, monitoring aims at the assessment of the necessity of reconstruction and the respective point in time. To establish objective criteria for a decision, indicators are introduced. These indicators should reflect all available information about the time-dependent structural behavior. Due to the fact that these indicators assess uncertain processes, they are obtained time-dependent and uncertain as well. The criteria for the assessment of the necessity of reconstruction are defined by requirements regarding the bearing capacity and serviceability.

One approach is to apply the time-dependent reliability. This indicator enables, e.g., the assessment of the damage state of a structure under consideration of longterm material behavior, the influence of modification processes, the consideration of conversions, etc. Advantageously, this indicator represents inherently a sophisticated measure to support the determination of the date of reconstruction. The time-dependent reliability is quantified by the fuzzy failure probability [7]

$$\tilde{P}_f(\tau) = \int_{\underline{x}|g_\tau(\underline{\tilde{s}},\underline{x})\leq 0} f_\tau(\underline{\tilde{s}},\underline{x})\, d\underline{x}. \tag{7}$$

An alternative indicator, to evaluate the structural state, is the robustness indicator. Robustness is determined, analogous to [12], by analyzing the influence of variations within the input parameters on the variations of the result parameters. According to [4], a structure is denoted as robust, if it result parameters are affected marginally by variations of input parameters. The time-dependent robustness of structures, in this approach considered for fuzzy processes only, is assessed by the indicator

$$I_R(\tau) = \frac{M_{\underline{x}}(\tau)}{M_{\underline{z}}(\tau)} = \frac{\sum_{k=1}^{n} M_{x_k}(\tau)}{\sum_{j=1}^{m} M_{z_j}(\tau)}. \tag{8}$$

In eq. (8) the influence of the uncertainty of n structural input parameters $\underline{\tilde{x}}(\tau)$ on the uncertainty of m structural responses $\underline{\tilde{z}}(\tau)$ is obtained. Thereby, the time-dependent parameters and results are discretized in time. An uncertainty measure M_x of a fuzzy number $\tilde{x}$ can be defined for instance on the basis of SHANNON's entropy

$$M_x : H_u = -\int_{x=-\infty}^{x=+\infty} \Big[\mu(x) \cdot \ln \mu(x) + \big(1-\mu(x)\big) \cdot \ln\big(1-\mu(x)\big)\Big] dx, \tag{9a}$$

as well as the zeroth moment

$$M_x : A = \int_{x=-\infty}^{x=+\infty} \mu(x)\, dx \tag{9b}$$

or the second central moment

$$M_x : V = \int_{x=-\infty}^{x=+\infty} (x-\bar{x})^2 \cdot \mu(x)\, dx \cdot \left(\int_{x=-\infty}^{x=+\infty} \mu(x)\, dx \right)^{-1} \tag{9c}$$

by means of the first moment

$$\bar{x} = \int_{x=-\infty}^{x=+\infty} x \cdot \mu(x)\, dx \cdot \left(\int_{x=-\infty}^{x=+\infty} \mu(x)\, dx \right)^{-1} \tag{10}$$

of the time-dependent fuzzy set $\tilde{x}(\tau)$.

3.2 Numerical simulation

If the uncertainty of the input parameters of a structural analysis is modeled with the aid of fuzzy random functions, the fuzzy stochastic analysis

$$F_{FSA} : \underline{\tilde{X}}(\underline{t}) \rightarrow \underline{\tilde{Z}}(\underline{t}). \tag{11}$$

is then to be solved. The fuzzy random functions (structural input parameters) $\underline{\tilde{X}}(\underline{t})$ are mapped onto the fuzzy random functions (structural responses) $\underline{\tilde{Z}}(\underline{t})$. As fuzzy functions and real random functions are special cases of fuzzy random functions, these uncertainty models are also incorporated in eq. (11).

To solve this problem numerically the fuzzy stochastic input parameters $\underline{\tilde{X}}(\underline{t})$ have to be discretized at points $\underline{t}_i$ in parameter space $\underline{\mathbf{T}}$. The fuzzy stochastic analysis has to be performed for every discrete input vector $\underline{\tilde{X}}_{t_i} = \underline{\tilde{X}}(\underline{t}_i)$ which leads to a response vector $\underline{\tilde{Z}}_{t_i} = \underline{\tilde{Z}}(\underline{t}_i)$.

For a fuzzy stochastic vector $\underline{\tilde{X}}$, eq. (11) can be rewritten by

$$\underline{\tilde{Z}} = \big(\tilde{Z}_1, \ldots, \tilde{Z}_j, \ldots, \tilde{Z}_m\big) = f\big(\tilde{X}_1, \ldots, \tilde{X}_k, \ldots, \tilde{X}_n\big) \tag{12}$$

whereas $f(.)$ represents the mapping model of the fuzzy stochastic analysis.

The fuzzy stochastic structural analysis bases on the bunch parameter representation of fuzzy random variables and is described in detail in [6, 9, 11]. Fuzzy random input variables $\tilde{\underline{X}} = \underline{X}(\underline{\tilde{s}})$ as well as fuzzy random result variables $\tilde{\underline{Z}} = \underline{Z}(\underline{\tilde{\sigma}})$ are expressed in dependency of fuzzy bunch parameter $\tilde{s}$ respectively $\tilde{\sigma}$. Therewith, eq. (12) is transformed into the mapping

$$\underline{\tilde{\sigma}} = \left(\tilde{\sigma}_1, \ldots, \tilde{\sigma}_j, \ldots, \tilde{\sigma}_{m_1}\right) = m\left(\tilde{s}_1, \ldots, \tilde{s}_k, \ldots, \tilde{s}_{n_1}\right) \tag{13}$$

Applying α-discretization [8] to the fuzzy bunch parameters, an optimization problem is solved in order to determine the α-level sets of the fuzzy bunch parameters $(\tilde{\sigma}_1, \tilde{\sigma}_2, ..., \tilde{\sigma}_{m_1})$. This algorithm is referred to as fuzzy analysis and described, e.g., in [6]. Each point of the input α-level sets leads to a stochastic analysis. Within the stochastic analysis, e.g., applying the Monte Carlo simulation, a deterministic fundamental solution $d(.)$ is processed repeatedly. Therewith a three-loop computational algorithm is constituted, Figure 4.

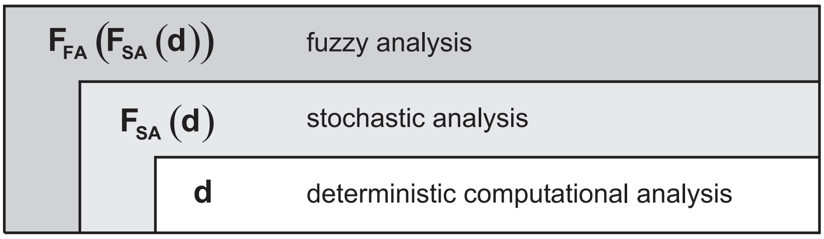


Figure 4: Fuzzy stochastic analysis F_{FSA}

The fuzzy analysis in combination with the Monte Carlo simulation is referred to as Fuzzy Monte Carlo simulation (FMCS), [11].

4 Example

The application of an uncertain numerical monitoring is demonstrated for a reinforced concrete two-field T-beam bridge (Figure 5). Under consideration of a time-dependent structural deterioration the safety level of the ultimate load as well as the robustness of the safety level are determined.

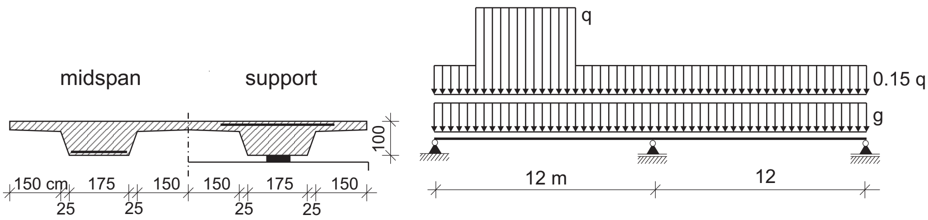
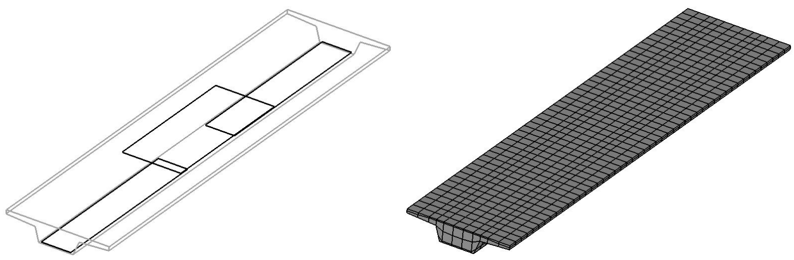


Figure 5: Geometry of reinforced concrete bridge

The bridge is computed with LS-DYNA and LS-PrePost. Thereby, the reinforcement in midspan and support is modeled by 232 two-dimensional shell elements, see Figure 6a. The concrete parts are modeled by 1536 three-dimensional solid elements, see Figure 6b. An elasto-plastic approach is applied for describing the material parameters.



(a) location of reinforcement (b) FE model

Figure 6: Model of the bridge in LS-PrePost

The results are basically affected by the concrete compressive strength f_c, the yield strength of reinforcement f_y, and the external loads q. Due to the fact that only rare and imprecise data are available, which impedes the specification of an unique stochastic model, the generalized uncertainty model fuzzy randomness is applied. Thereby, the parameters of the fuzzy stochastic distribution functions are modeled by fuzzy triangular numbers $\langle x_{l,\alpha=0}, x_{\alpha=1}, x_{r,\alpha=0} \rangle$, Table 1.

Table 1: Fuzzy random design parameters

		distribution			
		type	par 1	par 2	par 3
concrete compressive strength $[N/mm^2]$	f_c	log-normal	$\mu_x = 43$	$\sigma_x = \langle 2.5, 3.5, 4.5 \rangle$	$x_0 = 25$
yield strength of reinforcement $[N/mm^2]$	f_y	log-normal	$\mu_x = 540$	$\sigma_x = \langle 23, 33, 43 \rangle$	$x_0 = 450$
external loads $[kN]$	q	Gumbel	$a = 110.83$	$b = 16.67$	$n = 100$

The structure is subjected to its dead load g as well as the external loads q. Dynamic influences on the external loads are considered by the introduction of a statical vibration coefficient.

In numerical monitoring it is essential to capture time-dependent influences on the structure. An important one is the deterioration, for instance, as result of environmental exposures. However, the structural damage is affected by various influences that can not be determined exactly. In consequence, the deterioration is modeled by a fuzzy process

$$\tilde{d}_K(\tau) = d_K(\tau, \tilde{s}) = e^{-\int_0^\tau \chi(t,\tilde{s})dt} \tag{14}$$

which acts on the global stiffness matrix, Figure 7. Thereby, the fuzzy process $\chi(\tau, \tilde{s})$ is determined as

$$\chi(\tau, \tilde{s}) = \begin{cases} 0.015 \cdot e^{-0.1238 \cdot \tau \cdot \tilde{s}} & \text{if } \tau \leq \tau_1/\tilde{s} \\ 10^{-3} & \text{if } \tau_1/\tilde{s} < \tau \leq \tau_2 \\ 10^{-3} + \frac{1}{1.640.000}\left(e^{0.0323 \cdot \tau/\tilde{s}} - e^{0.0323 \cdot \tau_2/\tilde{s}}\right) & \text{if } \tau > \tau_2 \end{cases} \tag{15}$$

with $\tau_1 = 22$ and $\tau_2 = 100$ years and the bunch parameter $\tilde{s} = \langle 0.808, 1.0, 1.205 \rangle$.

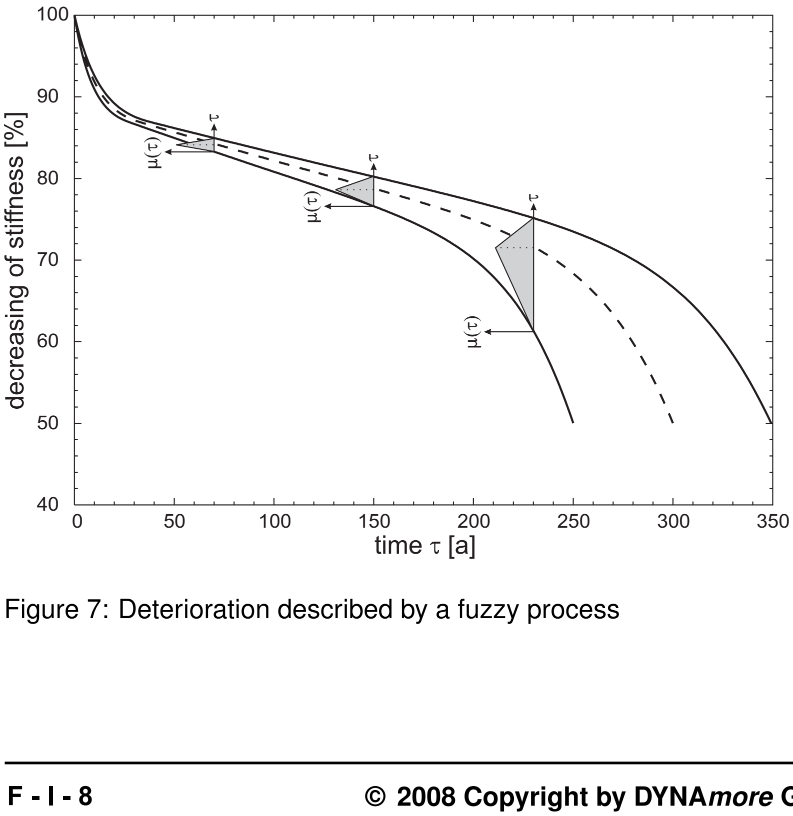
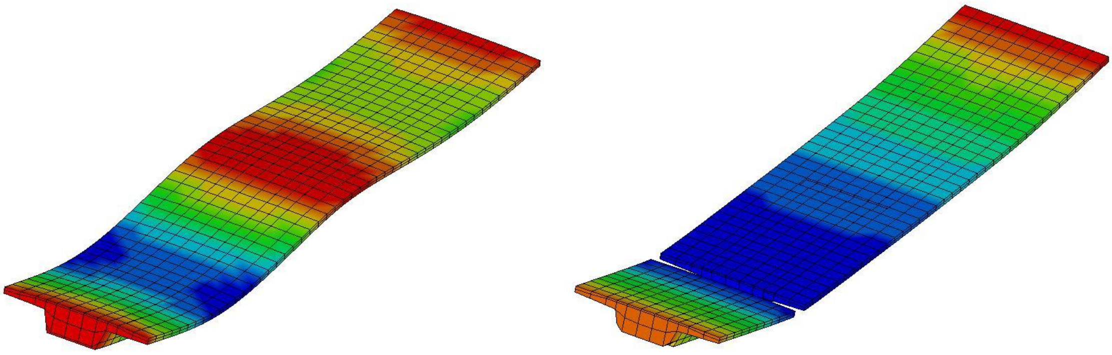


Figure 7: Deterioration described by a fuzzy process

The deterioration is modeled in an inverse way as load curve in the LS-DYNA keyword file. In this sense LS-DYNA is used for numercial simulation of the bridge over the lifetime, including collapse. In Figure 8 the deformed model is shown before and after exeeding serviceability, for a specific realization of f_c, f_y, and q.



(a) deformation before breakdown (b) collapse

Figure 8: Deformed FE model for specific realizations of the input parameters

A lifetime-orientated design aims among others on defining the lifetime. Therefore, an assessment of the structural reliability is performed by analyzing the fuzzy reliability index $\tilde{\beta}(\tau)$, Figure 9. Due to the time-dependence of input parameters the reliability alters over the lifetime. Introducing permissible limits of reliability measures such as the failure probability, e.g., predefined in building codes, a fuzzy lifetime $\tilde{L}$ can be determined. A reliable utilization of the structure in this example is prognosticated for $\langle 111, 149, 182\rangle$ years.

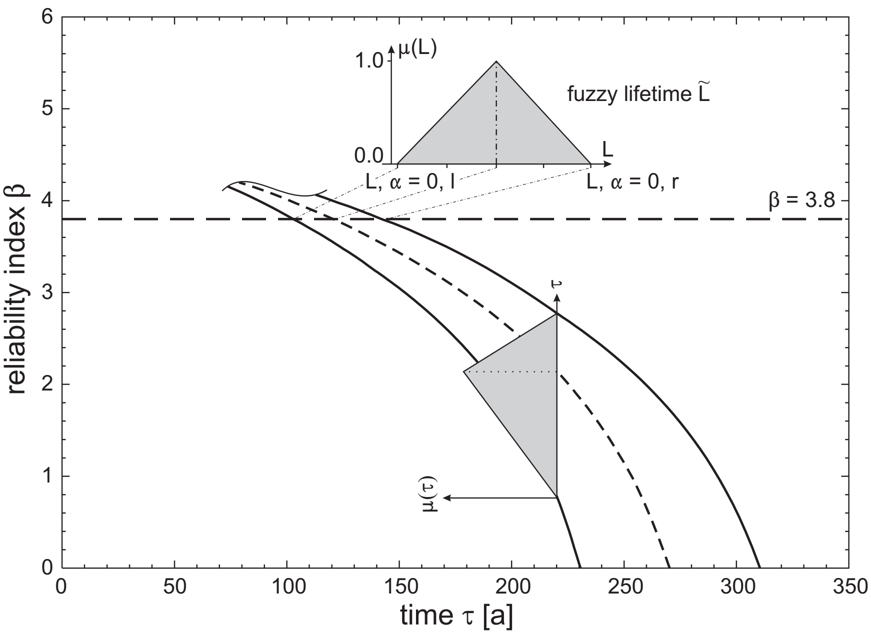


Figure 9: Fuzzy reliability index

Additionally quantifying the influence of the non-stochastic uncertainty the robustness of the reliability measure $\tilde{\beta}(\tau)$ is evaluated with the robustness measures $I_R(\tau)$ according to section 3.1, see Figure 10.

The structural indicators recorded with the aid of numerical structural monitoring facilitate the evaluation of the structural state at arbitrary points in time. The prediction of a lifetime enables the determination of a reliable service time.

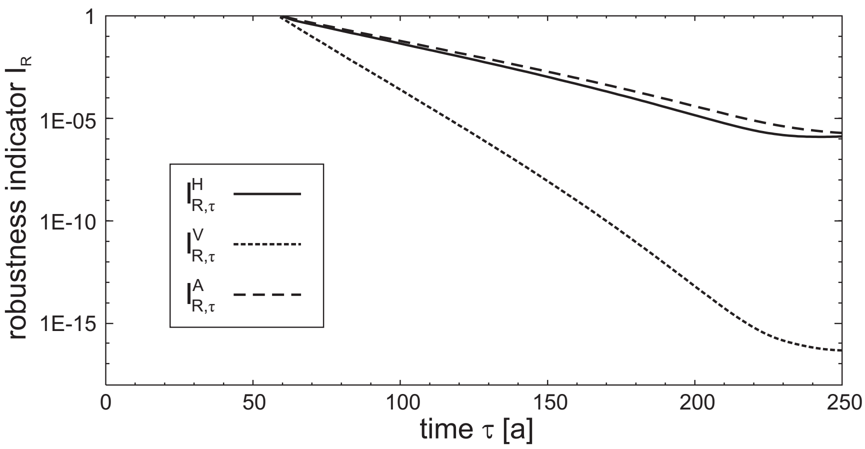


Figure 10: Robustness indicator

Acknowledgments

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