



XX	SimTech Cluster of Excellence	Universität Stuttgart Germany	Institut für Mecha Prof. DrIng. W. Eh		
	▷ What are multi-physics materials?				
Introduction Theory					
Numerical	foam	porous-media flow			
Simulation	→ the microstructural surface-coupled problems is more conveniently described through a macroscopic volume-coupled approach if the				
Application	microstructure is				
Examples	 unkown (e. g. in geomechanical problems) 				
Summary & Future Aspects	• or very complex (e. g	g. in biomechanical problems)			
	→ in multi-physics mate is driven by multiple m physical phenomena	rial the macroscopic behaviour utual interacting microstructura	al		
-3-					



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		Theoretical Fundamentals			
▷ I neory of Porous iviedia [Bowen, de Boer, Enlers, Lewis & Schre					
	• Saturated solid skeleton with (multi-component) pore-fluid content				
Introdu	ction	solid s e. g. soil,	keleton: φ^S		
Theory					
Numeri Treatm	cal ent	pore flue. g. wate	uid: $arphi^F = igcup_eta arphi^eta$ r, air, interstitial fluid	$ \label{eq:fluid_mixture} \begin{tabular}{lllllllllllllllllllllllllllllllllll$	
Simulat	ion	 Homogenisation process 	of multiphasi	c porous media	
Applica	tion	Representative	macroscale		
Example	es	Elementary Volume (REV)		micro-to-macro transition	
Summa Future	ry & Aspects		dv	$\rho^{\alpha R} := \frac{1}{V_m^{\alpha}} \int\limits_{V^{\alpha}} \rho_m^{\alpha} \mathrm{d} v_m^{\alpha}$	
			"homogenised model"	$ ho^lpha:=rac{1}{V_m}\int ho^lpha_m\mathrm{d}v^lpha_m$	
			dv^{s}	$n^{\alpha} := \frac{1}{1} \int \mathrm{d}v_m^{\alpha}$	
-5-		of the underlying microscale	dv'	$V_m \int_{V^{\alpha}} m$	

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	• Material independent balance eq	Juations
	Balance relations for the overall aggregate	Balance relations for the particular constituents
	momentum: $\rho \mathbf{\ddot{x}} = 0$ $\mathbf{r} + \rho \mathbf{u} \mathbf{v} \mathbf{x} = 0$	mass: $(\rho^{-})_{\alpha} + \rho^{-} \operatorname{anv} \mathbf{x}_{\alpha} = \rho^{-}$ momentum: $\rho^{\alpha} \stackrel{w}{\mathbf{x}}_{\alpha} = \operatorname{div} \mathbf{T}^{\alpha} + \rho^{\alpha} \mathbf{b}^{\alpha} + \hat{\mathbf{p}}^{\alpha}$
	$\mathbf{m}. \mathbf{o}. \mathbf{m}.: \qquad 0 = \mathbf{I} \times \mathbf{T} \rightarrow \mathbf{T} = \mathbf{T}^T$	m. o. m.: $0 = \mathbf{I} \times \mathbf{T}^{\alpha} + \hat{\mathbf{m}}^{\alpha}$
Introduction	$\begin{array}{ll} \text{energy:} & \rho \hat{\varepsilon} &= \mathbf{T} \cdot \mathbf{L} - \operatorname{div} \mathbf{q} + \rho r \\ \text{entropy:} & \rho \dot{\eta} &\geq \operatorname{div} \phi_{\eta} + \sigma_{\eta} = \operatorname{div} \left(-\frac{1}{\theta} \mathbf{q} \right) + \frac{1}{\theta} \rho r \end{array}$	$\begin{array}{lll} \text{energy:} & \rho^{\alpha} (\varepsilon^{\alpha})_{\alpha}^{\prime} &= \mathbf{T}^{\alpha} \cdot \mathbf{L}_{\alpha} - \operatorname{div} \mathbf{q}^{\alpha} + \rho^{\alpha} r^{\alpha} + \varepsilon^{\alpha} \\ \text{entropy:} & \rho^{\alpha} (\eta^{\alpha})_{\alpha}^{\prime} &= \operatorname{div} (-\frac{1}{\theta^{\alpha}} \mathbf{q}^{\alpha}) + \frac{1}{\theta^{\alpha}} \rho^{\alpha} r^{\alpha} + \hat{\zeta}^{\alpha} \end{array}$
Theory	 Resulting constraints and relation 	ns
Numerical Treatment	Specific constraints for total and direct production terms	Relations between total and partial quantities
Simulation	$\sum_{\alpha} \hat{\rho}^{\alpha} = 0$ $\sum_{\alpha} \hat{\mathbf{s}}^{\alpha} = 0 \text{with} \hat{\mathbf{s}}^{\alpha} = \hat{\mathbf{p}}^{\alpha} + \hat{\rho}^{\alpha} \mathbf{x}'_{\alpha}$	$\rho \mathbf{b} = \sum_{\alpha} \rho^{\alpha} \mathbf{b}^{\alpha}$ $\mathbf{T} = \sum_{\alpha=1}^{k} (\mathbf{T}^{\alpha} - \rho^{\alpha} \mathbf{d}_{\alpha} \otimes \mathbf{d}_{\alpha})$
Application Examples	$\sum_{\alpha} \hat{\mathbf{h}}^{\alpha} = 0 \text{with} \hat{\mathbf{h}}^{\alpha} = \hat{\mathbf{m}}^{\alpha} + \mathbf{x} \times \hat{\mathbf{s}}^{\alpha}$ $\sum_{\alpha} \hat{\mathbf{n}}^{\alpha} = \hat{\mathbf{n}}^{\alpha} + \hat{\mathbf{n}$	$\begin{split} \rho \varepsilon &= \sum_{\alpha} \rho^{\alpha} \left(\varepsilon^{\alpha} + \frac{1}{2} \mathbf{d}_{\alpha} \cdot \mathbf{d}_{\alpha} \right) \\ \mathbf{q} &= \sum_{\alpha} \{ \mathbf{q}^{\alpha} - (\mathbf{T}^{\alpha})^{T} \mathbf{d}_{\alpha} + \rho^{\alpha} \varepsilon^{\alpha} \mathbf{d}_{\alpha} + \frac{1}{2} \rho^{\alpha} \left(\mathbf{d}_{\alpha} \cdot \mathbf{d}_{\alpha} \right) \mathbf{d}_{\alpha} \} \end{split}$
Summary &	$\sum_{\alpha} e^{\mathbf{r}} = 0 \text{with} e^{\mathbf{r}} = \varepsilon^{\mathbf{r}} + \mathbf{p}^{\mathbf{r}} \cdot \mathbf{x}_{\alpha} + \rho^{\mathbf{r}} (\varepsilon^{\alpha} + \frac{1}{2} \mathbf{x}_{\alpha} \cdot \mathbf{x}_{\alpha})$ $\sum_{\alpha} \hat{\eta}^{\alpha} \ge 0 \text{with} \hat{\eta}^{\alpha} = \hat{\zeta}^{\alpha} + \hat{\rho}^{\alpha} \eta^{\alpha}$	$\begin{split} \rho r &= \sum_{\alpha} \rho^{\alpha} \left(r^{\alpha} + \mathbf{b}^{\alpha} \cdot \mathbf{d}_{\alpha} \right) \\ \rho \eta &= \sum_{\alpha} \rho^{\alpha} \eta^{\alpha} \end{split}$

Summary & Future Aspects

-6-

• Constitutive equations

- \circ Required to account for the $\mathit{closure\ problem}$ and to describe the $\mathit{physical\ response}$ of multiphasic materials
- \circ Derived from the entropy inequality in order to satisfy thermodynamical consistency \rightarrow depends on the investigated modelling approach

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57	Numerical Treatment				
	Discretisation				
	 Spatial discretisation: mixed finite elements (FE) [<i>Taylor & Hood</i> 1973] → satisfy LBB condition 				
Introduction	• Temporal discretisation, e. g.				
Theory	 o implicit <i>Euler</i> or <i>Newmark</i> scheme [<i>Newmark</i> 1959] o explicit methods (e. g. central-difference method, forward <i>Euler</i> method) 				
Numerical					
I reatment	Solution Procedure				
Simulation					
Application	• Material-model implementation into the in-house FE code PANDAS				
Examples	○ coupled multi-field solver				
Summary & Future Aspects	 o biject oriented e code o monolithic solution strategy o sequential code o command-lines interface o pre- and post-processing via third party tools (e. g. Cubit, Tecplot) 				
-7- [1]P	orous media Adaptive Non-linear finite-element solver based on Differential Algebraic System (www.get-pandas.com)				

















